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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

ANALYSIS OF THE FIELD ARTILLERY  
BATTALION ORGANIZATION USING  
A MARKOV CHAIN

by

Matthew A. Finlon

September, 1991

Thesis Advisor:

Samuel H. Parry

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Analysis of the Field Artillery Battalion Organization  
Using a Markov Chain

by

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Major, United States Marine Corps  
B.S., University of Washington, 1979

Submitted in partial fulfillment  
of the requirements for the degree of

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Department of Operations Research

## ABSTRACT

This thesis develops a model for comparing Marine Corps field artillery battalion organizations. It specifically examines the 3X8 and 4X6 direct support battalions. The status of the battalions are represented as continuous time, finite state, semi-Markov chains. The primary measure of effectiveness (MOE) for comparing the two structures is the long-run expectation of the number of guns in position. A set of APL programs manipulates the transition probability matrices and mean sojourn times. It then returns the long-run equilibrium probabilities and mean recurrence times for the states. Sensitivity analysis is conducted to explore the effects of changes in the transition probabilities and sojourn times.

## **THESIS DISCLAIMER**

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.



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## **I. INTRODUCTION**

### **A. BACKGROUND**

Throughout the history of modern warfare, few weapon systems have matched the ability of the field artillery to decide the outcome of battle. As warfare has evolved, so has the use of field artillery. The rapid improvement in large caliber cannon, coupled with rapid-firing machine guns, completely dominated the battlefield in World War I. The highly mobile and electronically sophisticated nature of modern warfare has created the need to develop new and more responsive methods of employing field artillery. Units must be organized so that they can provide continuous fires while keeping the flexibility to respond to rapidly changing tactical situations. Maximizing firepower while retaining flexibility is an old and continuing problem for field artillery planners. It is certain to remain so.

### **B. PURPOSE**

The U. S. Marine Corps has operated with a variety of artillery structures over the last twenty years. Changes in weapon systems, personnel availability, and perceived threat have all dictated changes to basic artillery organization. The number of maneuver units requiring artillery support has increased while the number of artillery units has decreased. Artillery battalions that are organized to support three maneuver elements are often tasked with the mission of supporting from four to six elements. Operation Desert Storm served to highlight this problem. As a result, the Marine Corps is currently evaluating new artillery weapon systems and structures.

The purpose of this research is to provide the Marine Corps with a decision making tool, in the form of a quick and easy to use program, to compare two artillery battalion organizations. Additionally, the model will enable the user to compare the different methods of employment. The inputs to the model are based on doctrine, field observations, and tactical experience. The basic nature of the inputs and output will allow its use by individuals with little or no experience in current modeling techniques. Most importantly, the model will provide a basic measure of artillery support available.

### C. OVERVIEW

This thesis develops a model, based on a finite state continuous time Markov chain, to compare two different field artillery battalions. The first is organized into three firing batteries equipped with eight howitzers while the second has four firing batteries of six howitzers each. Chapter II will offer an overview of artillery organization fundamentals and tactics. Although classically inaccurate, for purposes of this research the term "gun" will be used interchangeably with howitzer.<sup>1</sup>

Chapter III will present the methodology used to develop the analytical model with a basic example. The use of an analytical model allows for simple manipulation of the input data, increased sensitivity analysis, and easy to understand results. The methodology will be extended to the actual model in Chapter IV.

Analysis of the results is presented in Chapter V. Although the primary measure of effectiveness (MOE) is the long-run expectation of howitzers available for action,

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<sup>1</sup>Technically, a gun refers to a weapon with a high muzzle velocity and low angle of fire. Common usage, however, applies it to any cannon system.

other MOEs include long-run equilibrium probabilities and mean recurrence times for the states. Chapter VI will close with a summary and submit any conclusions.



## II. MARINE CORPS ARTILLERY FUNDAMENTALS

### A. GENERAL

Field artillery has proven to be an indispensable part of any modern army.

The mission of the field artillery (FA) is to destroy, neutralize, or suppress the enemy by cannon, rocket, and missile fire and to help integrate all fire support into combined operations. Success of this mission demands effective integration of FA fires into the fire support plan and the scheme of maneuver and swift, exact execution from the time a target is acquired until ordnance is delivered on target. [Ref. 1: p. 1-2]

The maneuver force commander or his authorized representative (usually the force artillery commander) defines unit responsibilities through the assignment of tactical missions.

### B. TACTICAL MISSIONS

Field artillery tactical missions delineate in detail the fire support responsibilities for the artillery unit. The artillery commander assigns tactical missions down to the battalion level. Each tactical mission contains seven fire support responsibilities. The assignment of tactical missions answers the following questions:

- From whom and in what priority does the unit answer *calls for fire*?
- What is the *zone of fire* of the unit?
- Is there a requirement to *furnish forward observation (FO) teams, artillery liaison teams and fire support coordinators or replacements for combat losses*?
- With which artillery unit does the unit establish *liaison*?
- With whom does the unit establish additional external *communications*?
- Who *positions* the unit?

- Who *plans the fires* of the unit? [Ref. 2: p. 1-2]

The field artillery commander must properly execute these responsibilities to fulfill his mission.

There are four standard tactical missions. They are:

- Direct support (DS)
- Reinforcing (R)
- General support (GS)
- General support-reinforcing (GSR)

This thesis will only discuss the comparison of artillery battalions assigned the mission of direct support. Some scenarios, however, will compare artillery battalions in the direct support role but receiving reinforcing fires.

A unit assigned the direct support mission must be immediately responsive to calls for fire from closely committed units. It provides forward observation teams, artillery liaison teams, and fire support coordinators to the supported maneuver units. The DS artillery battalion develops its own fire plan and its zone of action is that of the supported unit. Additionally, unless otherwise ordered by higher field artillery headquarters, the DS artillery battalion is free to determine its own positioning. This allows it the flexibility to maintain close and continuous support to its assigned maneuver element.

## **C. ORGANIZATION FOR COMBAT**

Artillery organizes for combat to provide the most effective fire support for the scheme of maneuver. The artillery commander accomplishes this by establishing a command relationship and through assignment of a tactical mission. To get the optimum organization for combat, he must properly balance the following fundamentals:

- Adequate fire support for committed close combat units.
- Weight to the main attack in the offense or additional strength to the most vulnerable area in the defense.
- Facilitation of future operations.
- Immediately available fire support with which the commander can influence the action.
- Maximum feasible centralized control.

Organizing for combat is an art as well as a science. The artillery commander must draw on his experience as well as that of his staff and correctly assess the current and future tactical situations. The organization of the artillery battalion can impact greatly on the commander's ability to use his artillery to its full effect.

## **D. BATTALION ORGANIZATION**

Artillery battalions in the Marine Corps are currently organized as either Direct Support (DS) or General Support (GS) battalions. Either battalion may be given the tactical missions stated earlier, however, the DS battalions are specifically organized for the DS mission while GS battalions are used primarily for the reinforcing, general support, or general support-reinforcing role. Batteries within DS battalions are

assigned personnel to act as forward observers and artillery liaison officers while GS battalions do not maintain these personnel within their Tables of Organization (T/O). This thesis will only explore different ways to organize DS battalions.

The two methods of organizing DS artillery battalions that are being given the most attention in the Marine Corps are:

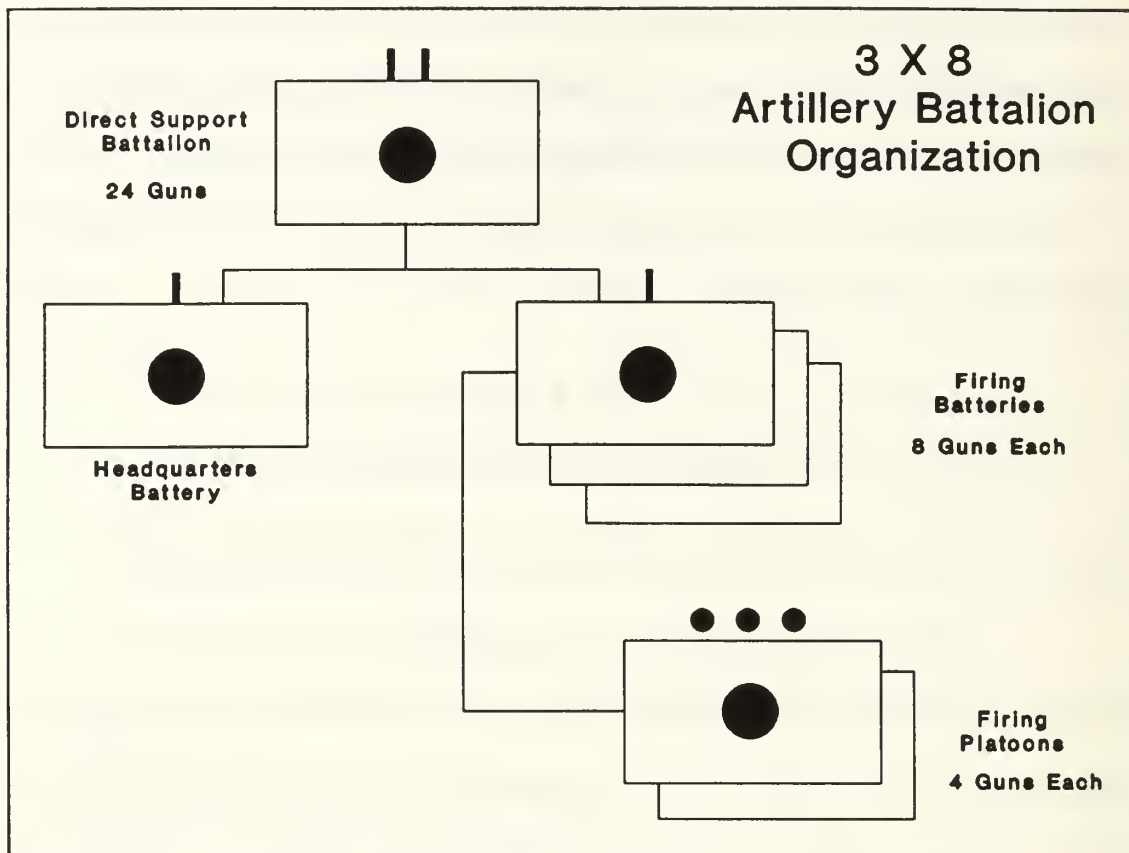
- A battalion with three batteries with eight howitzers each (3X8).
- A battalion with four batteries with six howitzers each (4X6).

These two organizations will form the basis for comparison in this thesis.

### **1. 3X8 Artillery Battalion Organization**

The 3X8 artillery battalion is the current organization in use by the Marine Corps and is shown in Figure 1. This organization was adopted in the early 1980's and was primarily a result of fielding the M198, 155mm howitzer. It replaced a 4X6 organization that comprised three batteries with six 105mm howitzers and one battery with six 155mm howitzers. Only the 105mm batteries had forward observation and liaison teams.

The main feature of the 3X8 structure is that it allows the battery to conduct split-battery operations. Split-battery operations give the battalion and battery commanders the option of moving the entire battery together or keeping one platoon of four howitzers in position to provide support while the second platoon moves to the next position. Once the second platoon occupies its position, the first platoon would be free to "leap-frog" forward.



**Figure 1. The 3X8 Artillery Battalion Organization**

While significantly increasing the batteries' firepower, there are several disadvantages with this formation. The major impact was in decreasing from four batteries to three. This resulted in a major loss of flexibility in the way the battalion commander could organize his unit. Marine Corps artillery battalions usually decentralize their calls-for-fire, meaning that when maneuver units need support they usually contact the battery that is supporting them directly. If the artillery battalion is supporting more than three maneuver battalions it must use more centralization (all calls-for-fire go to the artillery battalion fire direction center and are then passed to the batteries) or batteries are tasked with directly supporting several units. Either case results in an increased workload in the respective fire direction centers.

The size of the battery is larger under the 3X8 structure than the 4X6 organization. In addition to adding two howitzer sections with associated prime movers and crews, the number of support vehicles increased to meet the expanded logistics demands of the 155mm howitzer. The prime movers were upgraded from 2½-ton trucks to 5-ton trucks. The larger number and greater size of the vehicles and howitzers created a larger battery "footprint" or square footage occupied by the battery. This led to a need for more space on amphibious shipping and aircraft to transport the battalion or batteries.

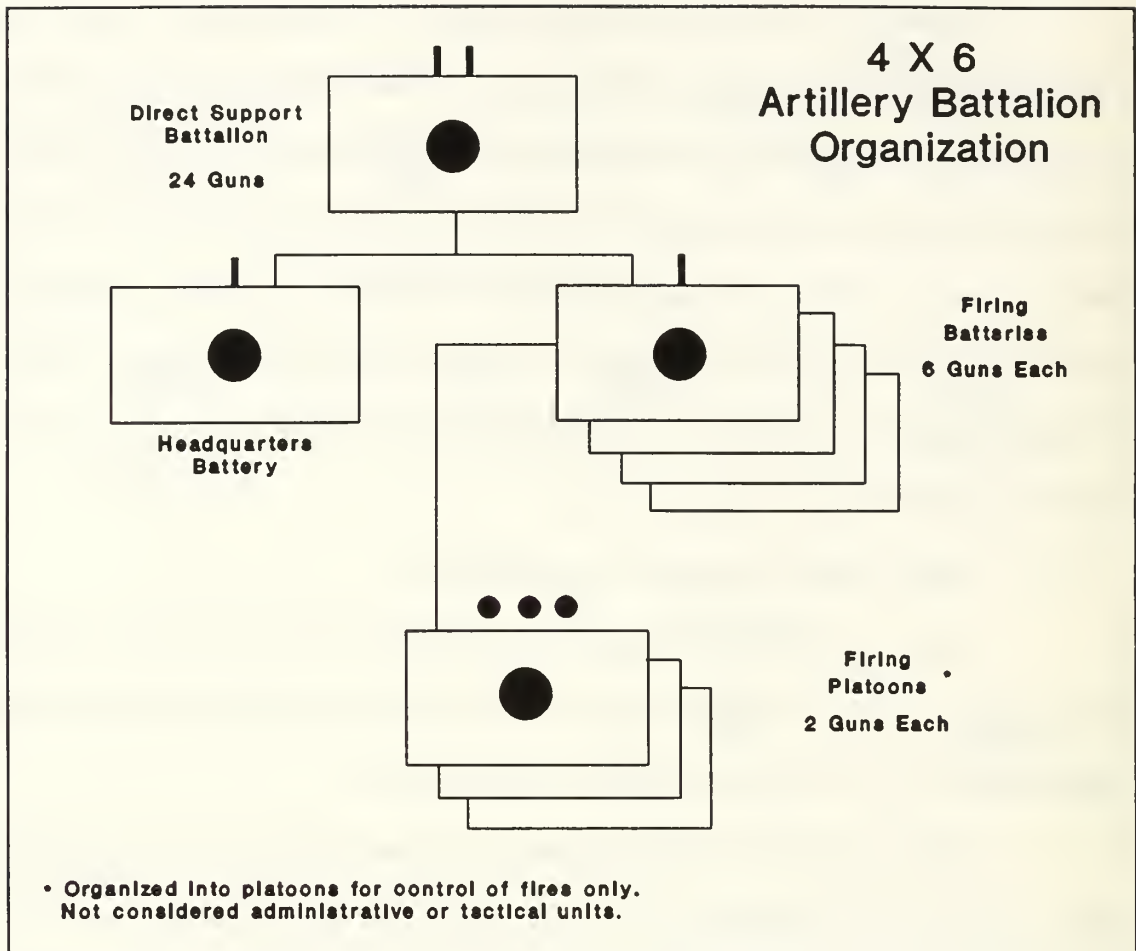
## **2. 4X6 Artillery Battalion Organization**

Many proponents are appealing for adoption of this structure. It differs from previous 4X6 formations in that all four batteries are equipped with the M198, 155mm howitzer and organized as illustrated in Figure 2.

While the battery in the 4X6 structure has less firepower (two less guns) than with the 3X8 organization, there is a great deal more flexibility within the battalion. Now the battalion can support four maneuver elements or the fourth battery can be used to weight the main attack by reinforcing another battery's fires. It can also be used by the battalion commander to provide general support or counterbattery fires.

Movement options are increased with the 4X6 structure. The fourth battery allows the battalion commander the ability to keep one entire battery moving while still providing significant fire support. Additionally, batteries under the old 4X6 formation proved that they could move by echelon (several howitzers moving while some stayed in position) and still provide continuous support.





**Figure 2. The 4X6 Artillery Battalion Organization**

## **E. MOVEMENT STRATEGY**

In combat, perhaps the greatest influence the field artillery battalion commander can have on the effectiveness of his unit is when he considers his movement options. He must always consider the current level of support needed as well as determine how to facilitate future operations. The analysis of the factors of METT (mission, enemy, terrain, and troops available) must be continuous.

The displacement method used depends on time available, scheme of maneuver of the supported unit, availability of reinforcing FA for continuous support, traffic conditions, and enemy activity. There are three ways to displace a battalion: by unit, echelon, and battery. [Ref. 1: p. 1-36]

Unit displacement is the fastest method with the battalion moving all its units together. It is normally used for long moves when the battalion does not have to provide supporting fires or when reinforcing fires are available. When moving by echelon, the battalion displaces one or more batteries and parts of the headquarters element. This allows some support to be continuously available. The 3X8 structure normally employs a variation of this by moving a platoon from each battery as well as a "jump" CP (command post) from the headquarters unit. Once in place, the remaining platoons displace along with the remainder of the headquarters elements. Movement by battery entails moving one battery at a time. This also allows for continuous fire support but greatly increases movement time for the battalion. This would normally be a series of short moves. It is anticipated that the 4X6 structure could make greater use of this method than the 3X8.

The method of displacement selected by the commander can have a telling effect on the ability of his unit to complete its mission. The structure of the artillery battalion can severely limit or increase the commander's options. Any discussion relating to artillery organizations must consider these advantages and disadvantages.

### III. METHODOLOGY

#### A. INTRODUCTION

There are several methods available to compare the attributes of the two artillery structures under consideration in this thesis. Artillery commanders and analysts need a basic methodology that provides meaningful MOEs and is easily manipulated. The final measurement of the quality of the model is whether it is used or not. The nature of artillery units lends itself to analysis by use of a continuous time Markov chain.

#### B. CONTINUOUS TIME MARKOV CHAINS

A continuous time Markov chain has the property that the future is independent of the past and depends only on the present state of the process. Additionally, the probability of moving from the present state to the next state is independent of the time the present state was entered. If the system is in state  $i$  at time  $s$ , the probability that it is in state  $i$  at time  $t + s$  is independent of  $s$  and depends only on the value of  $t$ . Similarly, the probability that the time required to transit out of state  $i$  is greater than  $t + s$ , given that the time to transit out of state  $i$  is greater than  $s$ , is independent of  $s$ . Again the probability depends only on the value of  $t$ . Thus, the probability distribution of the time required to transit a state never changes. This memoryless property is held by the exponential distribution. [Ref. 3: p. 582]

It can be said then that a continuous time Markov chain is a stochastic process denoted by  $\{X(t)\}$ , where  $t \geq 0$  takes on values  $0, 1, \dots, M$ . This process has the following properties:

- Each time the process enters a state  $i$ , the amount of time it spends in that state before transiting to a different state has an exponential distribution with mean sojourn time  $1/\nu_i$ .
- When leaving state  $i$ , the process moves to a state  $j$ , with probabilities  $p_{ij}$ , where the  $p_{ij}$  satisfy the conditions

$$p_{ii} = 0, \quad \text{for all } i,$$

and

$$\sum_{j=0}^M p_{ij} = 1, \quad \text{for all } i.$$

- The next state visited after transiting from state  $i$  is independent of the time spent in state  $i$ . [Ref. 3: p. 582]

The above definition hints that only three inputs are required for the continuous time Markov chain. First, the state space must be defined. The state space is merely a collection of categories that the process being examined can occupy. For an artillery unit, the state space may consist of two states: being in position or conducting a movement. Secondly, all the values for  $p_{ij}$ , the probability of transiting from state  $i$  to state  $j$  must be determined. Lastly, the sojourn parameters,  $\nu_i$  are needed for all the states. The sojourn parameter  $\nu_i$  is known as the intensity of passage, given that the Markov chain is in state  $i$ . Also,  $\alpha_{ij}$  is called the intensity of transition from state  $i$  to state  $j$ . The relationship between the intensity of passage and the intensity of transition is

$$v_i = \sum_{j=0, j \neq i}^M \alpha_{ij}.$$

The assumption of exponentially distributed sojourn times can be made even for nonexponential waiting times as long as the means for both distributions are equal [Ref. 4]. This greatly simplifies the calculations and requires no information about the distribution of sojourn times other than the means.

### C. MATHEMATICAL FORMULATION

Once the inputs are known, the computations consist of matrix manipulation and solving simultaneous equations.

#### 1. Infinitesimal Matrix

The transition rate from state  $i$  to state  $j$  is calculated from the relationship

$$\alpha_{ij} = v_i p_{ij}, \quad i \neq j.$$

Also

$$\alpha_{ii} = -v_i = - \sum_{j=0, j \neq i}^M \alpha_{ij}.$$

This relationship will be used to balance the flow into a state with the flow out of a state. It is used to comprise part of the infinitesimal matrix

$$A = \begin{vmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2n} \\ \dots & \dots & \dots & \vdots \\ \dots & \dots & \dots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \dots & \alpha_{nn} \end{vmatrix}.$$

## 2. Limiting Distribution

The long-run equilibrium probability for state  $j$  is

$$\lim_{t \rightarrow \infty} P_{ij}(t) = \pi_j.$$

Since  $\nu_j$  is the rate at which the process leaves state  $j$  given that it was in state  $j$ , the long-run rate at which it leaves state  $j$  must be  $\pi_j \nu_j$ . To maintain balance in the system, the long-run rate out of a state must equal the long-run rate into the state. Therefore, the long-run equilibrium probabilities are determined from the equation

$$\pi_j \nu_j = \sum_{i=1, i \neq j}^n \pi_i \alpha_{ij}, \quad j=1, 2, \dots, n.$$

This relationship along with the property

$$\pi_1 + \pi_2 + \dots + \pi_n = 1,$$

form the *balance equations*. The balance equations can be represented in the matrix form



$$\pi A = [ \pi_1 , \pi_2 , \dots , \pi_n ] \begin{vmatrix} \alpha_{11} & \dots & \alpha_{1n} \\ \alpha_{21} & \dots & \alpha_{2n} \\ \dots & \dots & \dots \\ \alpha_{n1} & \dots & \alpha_{nn} \end{vmatrix} = 0 .$$

### 3. Matrix Manipulation

The long-run equilibrium probabilities can be solved as a set of simultaneous equations. The formula for the multiplication of the transpose

$$A^t \pi^t = 0 ,$$

can be used with an APL program to quickly solve for the  $\pi_i$  values. The mean recurrence times, or how often the process returns to state  $i$  is computed from the formula

$$T_i = \frac{1}{\pi_i v_i} .$$

### D. OUTPUT

The methodology is useful in providing the long-run equilibrium probabilities and mean recurrence time for each state. The first is useful because it allows the commander to determine the percent of time he can expect to be in a certain state. This can be important if the unit is trying to avoid certain states or stay longer in

other states. The mean recurrence time may be needed for planning purposes such as resupply.

### E. EXAMPLE

To illustrate the methodology, an example is given. An artillery battery has a state space consisting of the following three states:

- State 1: Battery in position.
- State 2: Battery conducting short move.
- State 3: Battery conducting long move.

The probabilities of transiting between the states are given in Table 1.

TABLE 1. TRANSITION PROBABILITY MATRIX

STATE	1	2	3
1	0	.4	.6
2	1.0	0	0
3	1.0	0	0

The mean sojourn times in minutes for the states are given in Table 2.

TABLE 2. MEAN SOJOURN TIMES

STATE	1	2	3
TIME	240	60	180

The infinitesimal matrix is then computed. For example

$$\alpha_{11} = -\frac{1}{240}$$

and

$$\alpha_{12} = \frac{1}{240} \times 0.4.$$

The equations for the equilibrium distribution are

$$\begin{aligned} .004167\pi_1 &= .0167\pi_2 + .00555\pi_3, \\ .0167\pi_2 &= .001667\pi_1, \\ .00555\pi_3 &= .0025\pi_1, \\ \pi_1 + \pi_2 + \pi_3 &= 1. \end{aligned}$$

Solving the system of simultaneous equations yields

$$\pi_1 = .645, \quad \pi_2 = .291, \quad \pi_3 = .064.$$

Computing the mean recurrence times gives

$$T_1 = \frac{240}{.645} = 372, \quad T_2 = \frac{60}{.291} = 206, \quad T_3 = \frac{180}{.064} = 2812.$$

The commander now knows that he is spending 64.5 percent of his time in position and very little of his time doing long moves. Additionally, he knows how often he will return to a state.

## IV. MODEL DEVELOPMENT

### A. MEASURE OF EFFECTIVENESS

There are two measures of effectiveness (MOEs) that have typically been used to measure the degree of support an artillery unit has given. They are:

- Number of rounds fired in a day.
- Number of missions fired in a day.

While the first may be helpful to logisticians for planning purposes, neither MOE was an accurate portrayal of the degree of support the artillery unit was *capable* of. The manner in which a target was engaged is more important than to fire a lot of rounds and missions. Due to surprise, the initial volley fired on a target is usually the most destructive (assuming the rounds hit the target!). The enemy can take defensive measures to avoid effects from subsequent volleys. For this reason, depending on target type and posture, 48 rounds fired from 24 guns will probably be more destructive than 60 rounds fired from 12 guns. The first mission would have two volleys with 24 rounds per volley and the second would have five volleys with 12 rounds per volley. Therefore, total number of rounds fired is not a desirable MOE.

Using total number of missions fired as an MOE is even less defensible. Again it is a question of quality versus quantity. The effects of the missions are much more important than the number of missions fired.

A more accurate MOE to describe the level of support an artillery unit can give can be designed. The more howitzers the commander has in position, the more

engagement options he has. He can choose to fire on a target with all available weapons and maximize surprise, or fire a portion of his weapons and keep the others hidden from enemy counterbattery fire. The ultimate goal of every artillery commander, therefore, should be to maintain the maximum number of weapons in position ready to fire. By doing so he ensures that he can fire the maximum number of missions with the maximum number of rounds (the two previous MOEs) while doing it in the *most efficient and effective manner*. In other words, he gets quantity and quality.

A howitzer that is unable to shoot for any reason is useless. The movement of the guns contributes the most to the nonavailability of artillery. A good commander will always strive to minimize this downtime within the constraints of the situation in which he is operating in.

If given a choice of two artillery organizations, the commander should choose the one that through its structure obtains the largest *long-run expectation of guns in position ready to fire*. This will be the MOE used in this thesis to compare the two artillery organizations. As stated earlier, not only does this MOE address the question of quantity but also takes up the issue of quality of fires. The model will compute this MOE for the two structures in a variety of scenarios.

## **B. STATE SPACE**

As stated in the previous chapter, a state space is merely a collection of categories that the entity to be examined can occupy. Developing a state space can be difficult. Too few states may not accurately depict the dynamics of the system while too many states may prove to be untractable. For this model there are two levels of

state spaces that must be examined: the state space for the batteries and the state space for the battalions.

The state of a battalion at any point in time is a reflection of the different states the batteries are in at that same time. For example, a battery may occupy either a state of being in position or of moving. The state of a battalion with three batteries could be:

- All batteries in position.
- All batteries moving.
- One battery in position, two batteries moving.
- Two batteries in position, one battery moving.

Thus, for this scenario, there are two battery states but four battalion states. The number of states in the battery state space should be kept to a minimum. Otherwise, the battalion state space will grow very quickly.

The discussion on MOEs mentioned that movement of units was the largest contributor to nonavailability of weapons. The structure of an artillery battalion and its organic batteries has a significant effect on the manner in which it moves. Therefore, the state spaces developed for this model concentrate on battery moves. All actions conducted incident to moving e.g., displacing and emplacing the howitzers, are included in the movement times. Moves are separated into long (greater than 15 kilometers) and short (less than 15 kilometers) distances.



## 1. 3X8 Artillery Battery and Battalion State Space

The state space for the eight gun artillery battery organization consists of five states. They are:

- Eight guns in position.
- Eight guns conducting long move.
- Eight guns conducting short move.
- Four guns in position, four guns conducting long move.
- Four guns in position, four guns conducting short move.

The state space diagram is shown in Figure 3.

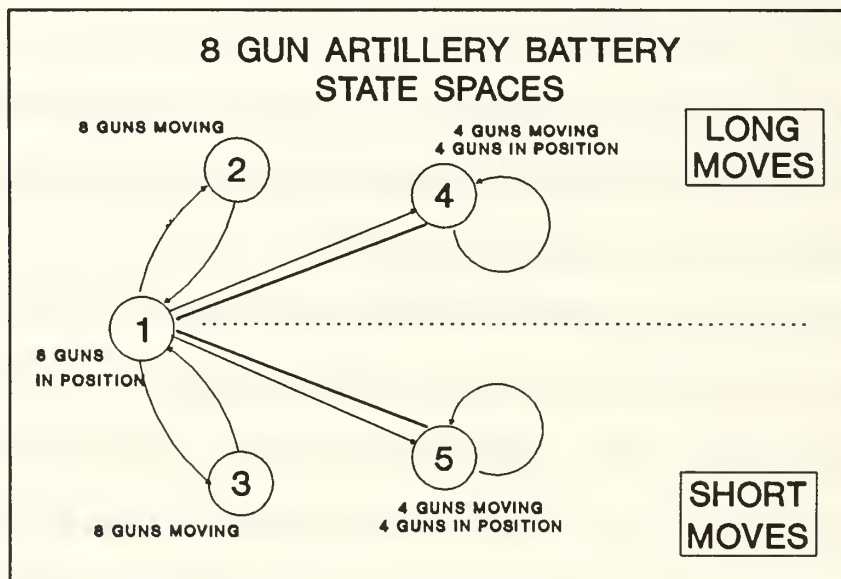


Figure 3. The 3X8 Artillery Battery State Space

When the battery is in state 2 or state 3 (eight guns moving) it returns to state 1 with probability of 1.0. Operating in state 4 and state 5, the battery transitions back to state 1 with probability of 0.5 or returns to its original state with probability of 0.5. This equates to the first platoon moving (state 4), then the second platoon

moving (remaining in state 4), and then all eight guns together (state 1). Thus, half the time the battery is in state 4 or state 5 it returns to that state and half the time it returns to state 1. Strictly speaking, by allowing the battery to return to the state it is in violates the second property of the continuous time Markov chain ( $p_{ii} = 0$ ). The model was initially run two ways: first as shown in the diagram and then without allowing it to return to the same state but doubling the sojourn time for state 4 and state 5. The long-run nature of the model causes the results from both runs to be identical. Therefore, because the diagram most accurately depicts the split-battery concept it is the model that was retained.

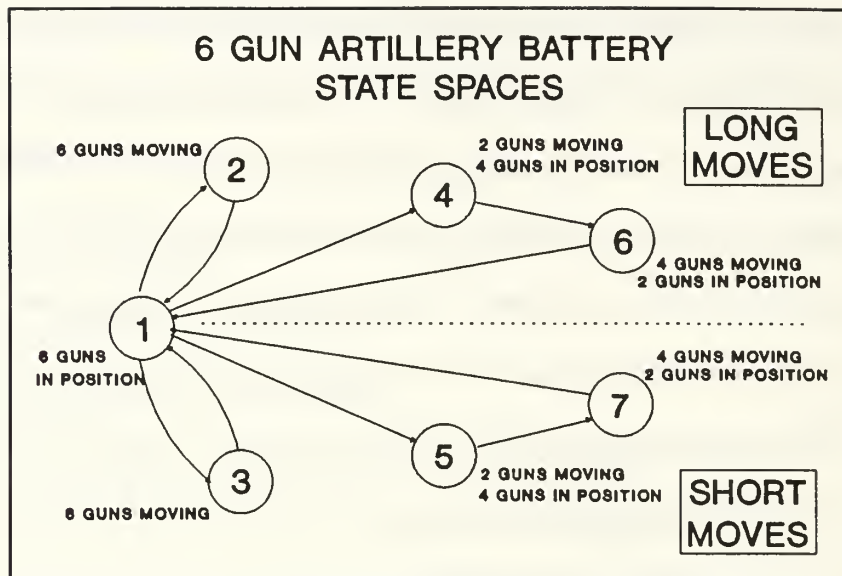
The possible combinations for the three batteries lead to 35 possible states for the battalion. They appear in Appendix A.

## **2. 4X6 Artillery Battery and Battalion State Space**

The state space for the six gun artillery battery organization consists of seven states. They are:

- Six guns in position.
- Six guns conducting long move.
- Six guns conducting short move.
- Two guns conducting long move, four guns in position.
- Two guns conducting short move, four guns in position.
- Four guns conducting long move, two guns in position.
- Four guns conducting short move, two guns in position.

The state space is shown in Figure 4.



**Figure 4. The 4X6 Artillery Battery State Space**

The six gun battery has two extra states because it lacks the symmetry of the eight gun battery. The batteries in a 4X6 battalion are organized into three platoons of two guns each. This structure is for control of fires primarily and is not an administrative or tactical configuration. The two gun and four gun split was chosen because in the past, many six gun batteries operated in this fashion as attachments to battalion landing teams (BLTs). It is conceivable, however, that the battery could split into two, three gun sections and have a state space similar to the eight gun batteries. State spaces 2, 3, 6, and 7 all return to state 1 with probability of 1.0. States 4 and 5 transition to 6 and 7 respectively with probability 1.0. In both the eight gun and six gun batteries, the only transition probabilities that are subject to change are those associated with transits *from* state 1.

The combinations for the four batteries results in 210 possible states for the battalion. They appear in Appendix B.

## **C. TRANSITION PROBABILITIES**

As mentioned earlier, the only transition probabilities that the commander can affect are those associated with leaving a position. Once a move is started, a battery must eventually return to a position with a probability of 1.0. Transition probability matrices were devised for different scenarios and are shown in Appendix C. The inputs for the matrices were based on the following:

- Interviews with Operation Desert Storm veterans. [Ref. 5]
- Operational journals and radio logs. [Ref. 6]
- Author's experience.

## **D. MEAN SOJOURN TIMES**

The mean sojourn times for the model are presented in Appendix D. The sources for the inputs are the same as for the transition probabilities.

## **E. COMPUTATIONS**

The computations were performed by a series of APL programs run on a mainframe computer. The programs are listed in Appendix E. The programs are interactive to allow for the mean sojourn times and transition probabilities from state 1 to be varied according to the scenario. Results are instantaneous. Results given are:

- Long-run equilibrium probabilities (probability of being in a state) for each battery configuration.
- Mean recurrence time (how soon a battery can be expected to return to a state once it leaves it) for each battery configuration.

- Long-run equilibrium probabilities for each battalion configuration (States with equal number of guns in position are combined resulting in 13 states for 4X6 and seven state for 3X8).
- Long-run expectation of guns in position for each battalion configuration (primary MOE). Computed by multiplying the long-run equilibrium probabilities for each state by the number of guns in position for that state and adding together.

## F. ASSUMPTIONS

In order to simplify the model several assumptions had to be made. First it is assumed that over the course of the operation none of the howitzers are removed from action for any reason other than movement. While this possibility could be added to the model e.g., a state could be five guns in position and one gun in maintenance, it is easy to see how quickly the battalion state spaces would grow. Additionally, the structure of the battalions would have little effect on the loss of howitzers due to breakdowns, personnel loss, etc.

This model is concerned with isolating the differences in movement methods for the two structures. Details such as vehicle breakdowns, loss of material, and casualties are better handled through more complex simulations.

The model also assumes that an operation would be of sufficient length to allow the long-run equilibrium probabilities to be valid. Again, as we are making this assumption for both organizations, the model results should be sound.

## **V. ANALYSIS**

### **A. SCENARIOS**

The model was run for five types of operations. These operations are representative of the majority of situations in which the artillery may be expected to employ. Unless specifically mentioned, all scenarios can be expected to be daytime operations.

#### **1. Movement to Contact**

The movement to contact operation usually involves a significant level of uncertainty. The purpose of this operation is to gain or regain contact with the enemy and to develop the situation sufficiently to determine whether a hasty or deliberate attack is in order [Ref. 1: p. 6-5]. Artillery units should plan for many moves and hasty attacks.

#### **2. Deliberate Attack**

The deliberate attack is conducted when an apparent enemy weakness has not been found. The plan of attack is more detailed and there is more time for planning purposes. Artillery units will be positioned well forward. They can expect to stay in position longer in order to provide continuous support to the attacking maneuver elements.

#### **3. Exploitation**

Exploitation is an operation undertaken to follow up success in the attack. It is a series of movements to contact and hasty attacks. All are conducted with two



overriding requirements--speed and violence [Ref. 1: p. 6-6]. Artillery units should plan for frequent moves. If opposition is light, units may move great distances in short periods of time.

#### **4. Defense of the Main Battle Area (MBA)**

Artillery will be primarily used to blunt the enemy attack, support counterattacks, and fire counterbattery missions. Movements will tend to be short moves for survivability and may be frequent in nature.

#### **5. Delay**

Delaying actions are used to gain time to establish the defense, cover defending and withdrawing units, to protect the flank of a supported element, and to participate in an economy-of-force effort [Ref. 1: p. 6-10]. Artillery will remain mobile throughout the operation. Moves will be a combination of short and long moves depending on the withdrawal schedule.

The model was run for each battalion structure under the five basic scenarios. Additionally, the model was run for the movement to contact and exploitation operations with reinforcing artillery. Reinforcing artillery covers the DS battalion's sector of responsibility thereby allowing the DS battalions to conduct unit movements more often. The movement to contact and exploitation phases (no reinforcing artillery) were also run for a nighttime scenario.

## B. OUTPUT

### 1. Procedures

The data from the transition probabilities matrices and mean sojourn time matrices for each scenario were entered into the APL programs. The model produced the output described in Chapter IV, Section E of this thesis.

The data for the batteries within the battalion were aggregated by the model to produce the battalion MOE. For example, if state 1 for a battery was six guns in position and state 1 for the battalion was three batteries all in position, the long-run equilibrium probability for battalion state 1 would equal the long-run equilibrium probabilities for the batteries (for battery state 1) multiplied by each other. Expressed mathematically,

$$BN\pi_1 = BTRY_A\pi_1 \times BTRY_B\pi_1 \times BTRY_C\pi_1$$

where

$$BN\pi_1 = \text{Long-Run Equilibrium Probability} \\ \text{for Battalion State 1,}$$

and

$$BTRY_A\pi_1 = \text{Long-Run Equilibrium Probability} \\ \text{for Battery A, State 1.}$$

The battalion long-run equilibrium probabilities were then aggregated. Table 3 shows how the battalion states for the 4X6 structure were combined, based on how many guns were in position for that state.

TABLE 3. AGGREGATED 4X6 BATTALION STATES

GUNS AVAIL ABLE	STATES WITH EQUAL GUNS AVAILABLE
0	2, 3, 15, 21, 56
●	18, 19, 24, 25, 93, 94, 108, 109
4	16, 17, 22, 23, 59, 60, 63, 64, 91, 92, 100, 106, 107, 115, 151, 166, 204
6	14, 20, 39, 40, 45, 46, 86, 96, 97, 98, 99, 101, 111, 112, 113, 114, 154, 157, 169, 172, 181, 182, 202, 203
8	6, 7, 43, 49, 57, 58, 61, 62, 70, 89, 90, 95, 104, 105, 110, 121, 136, 146, 147, 152, 153, 155, 156, 161, 162, 167, 168, 170, 171, 178, 179, 180, 207, 208, 209, 210
10	41, 42, 47, 48, 87, 88, 102, 103, 123, 124, 126, 127, 138, 139, 141, 142, 150, 159, 160, 165, 174, 175, 176, 177, 183, 184, 194, 197, 205, 206
12	27, 28, 33, 34, 38, 44, 50, 51, 66, 67, 68, 69, 71, 122, 125, 130, 137, 140, 145, 148, 149, 158, 163, 164, 173, 185, 187, 188, 190, 191, 192, 193, 195, 196
14	30, 31, 36, 37, 74, 75, 78, 79, 116, 117, 128, 129, 131, 132, 143, 144, 186, 189, 200, 201
16	4, 5, 29, 35, 54, 55, 65, 72, 73, 76, 77, 85, 119, 120, 134, 135, 198, 199
18	8, 9, 26, 32, 81, 82, 83, 84, 118, 133
20	12, 13, 52, 53, 80
22	10, 11
24	1

For example, battalion states 8, 9, 26, 32, 81, 82, 83, 84, 118, 133 are all states that have 18 guns in position. The model adds the long-run equilibrium

probabilities for these states to get the probability of being in a pseudo-state of 18 guns in position. Without this aggregation, there would be a collection of very small long-run equilibrium probabilities. Now we have seven probabilities for the 3X8 structure (0, 4, 8, 12, 16, 20, or 24 guns up) and thirteen for the 4X6 organization (0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, or 24 guns up). The reduction from 210 states to 13 is significant for the 4X6 battalion and makes the comparisons easier.

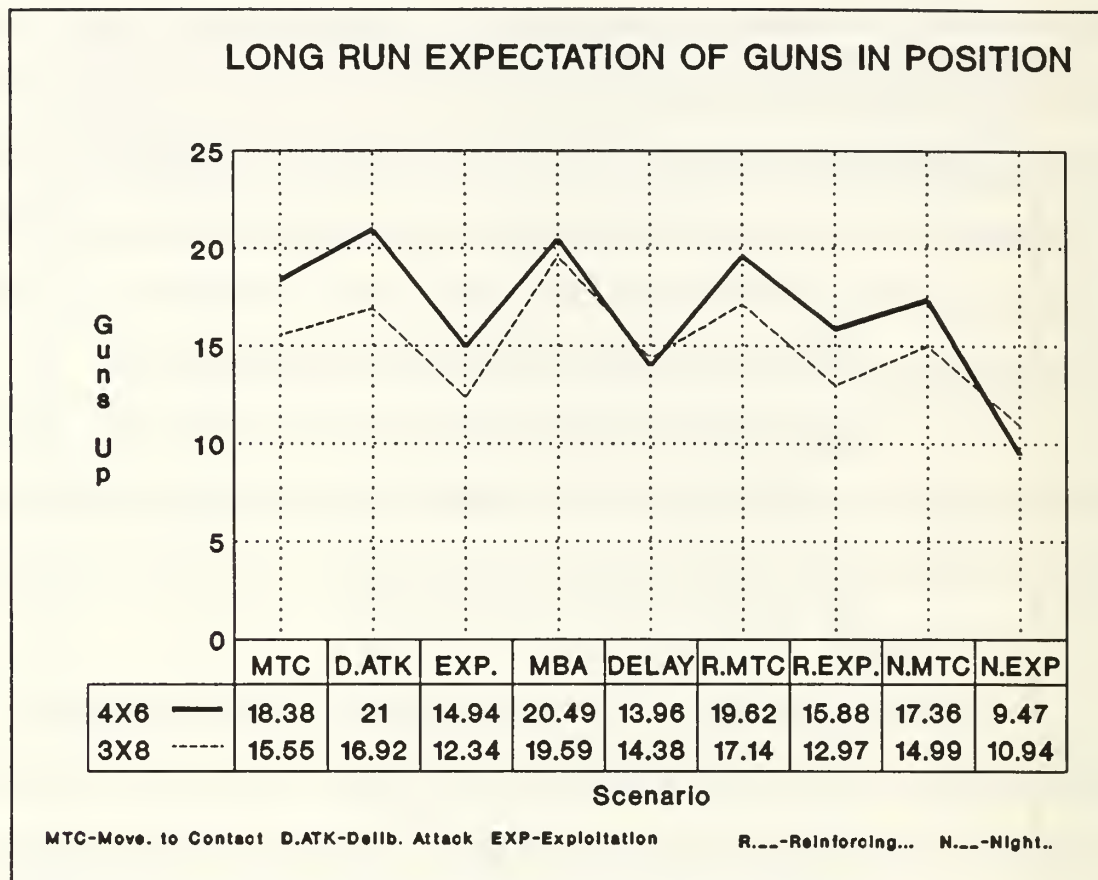
The primary MOE of expected number of guns in position was computed by multiplying the long-run equilibrium probabilities for the pseudo-states mentioned above, by the number of guns in position for that pseudo-state, and adding together.

## **2. Results**

The MOE results for the different scenarios are presented in Figure 5. For the majority of the cases the 4X6 battalion has the larger long-run expectation of guns in position. It suggests that for the values given to the model, the 4X6 structure will usually lead to having more guns in position. As an added feature, the commander can compare the long-run equilibrium probabilities for the battalion pseudo-states. An example of this type of output for the 4X6 battalion is illustrated in Figure 6.

## **C. SENSITIVITY ANALYSIS**

There are two inputs that can be manipulated for sensitivity analysis: sojourn times and transition probabilities. The commander can get a good feel for the effect each of these factors has on the primary MOE. He can then use this information for planning purposes. The following sensitivity analysis was run with the 4X6 battalion in a daylight movement to contact.

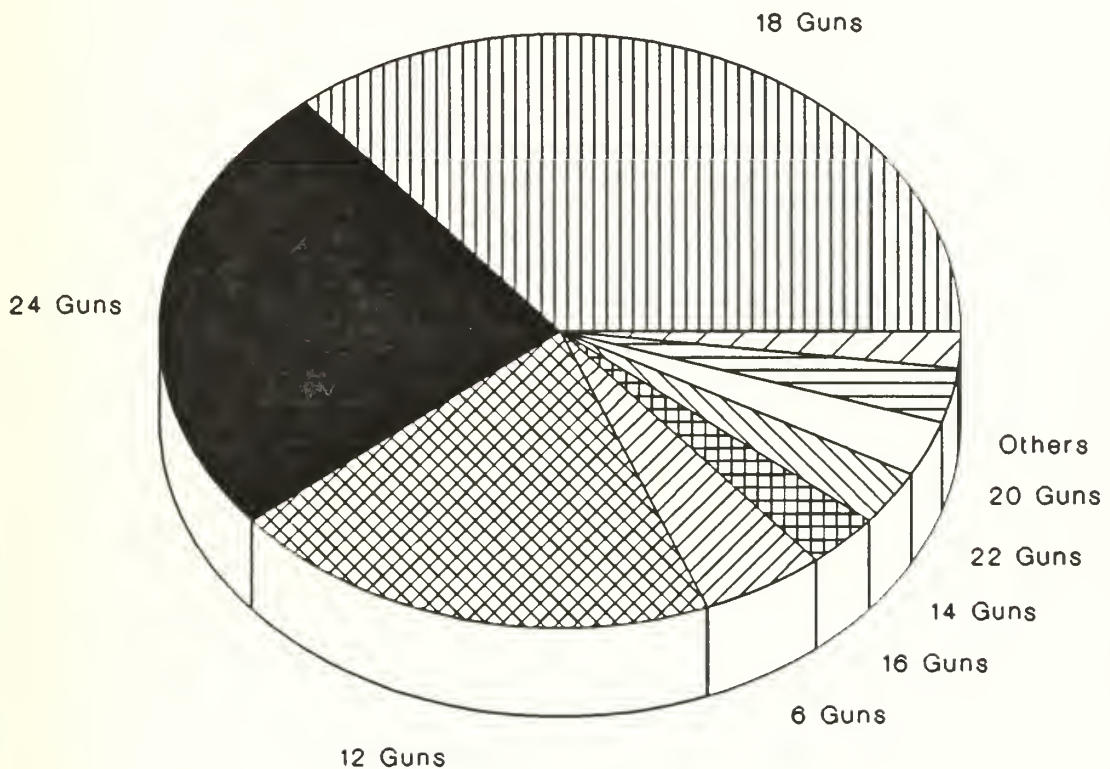


**Figure 5. The Long-Run Expectation of Guns in Position**

Figure 7 shows the effects of varying the sojourn time for state 1 (six guns in position) for a battery in a 4X6 battalion. The sojourn times for all states other than state 1 were fixed as were the transition probabilities. This is a realistic assumption as the commander has little control over the time it takes units to move. He can, however, have a profound effect on the amount of time a battery stays in a position. As the figure shows, when the sojourn time increases, the number of guns in position increases. While the increase is rather dramatic at first, the benefits derived from staying in position longer decrease as the sojourn times get larger. A commander could use this information to decide at what point he wants to trade off obtaining the maximum possible number of guns in position with the dangers of staying in one



## LONG RUN EQUILIBRIUM PROBABILITIES 4X6 Battalion Pseudo-States



**Figure 6. The 4X6 Battalion Pseudo-State Long-Run Equilibrium Probabilities**

position too long.

The commander may also desire to know how the proportion of battery moves to platoon moves and the proportion of short moves to long moves affect the MOE. Figure 8 shows that for this particular scenario, moving by platoons is slightly better than moving by battery with regards to the MOE and that short moves result in a slightly larger number of guns in position. This information can be weighed against



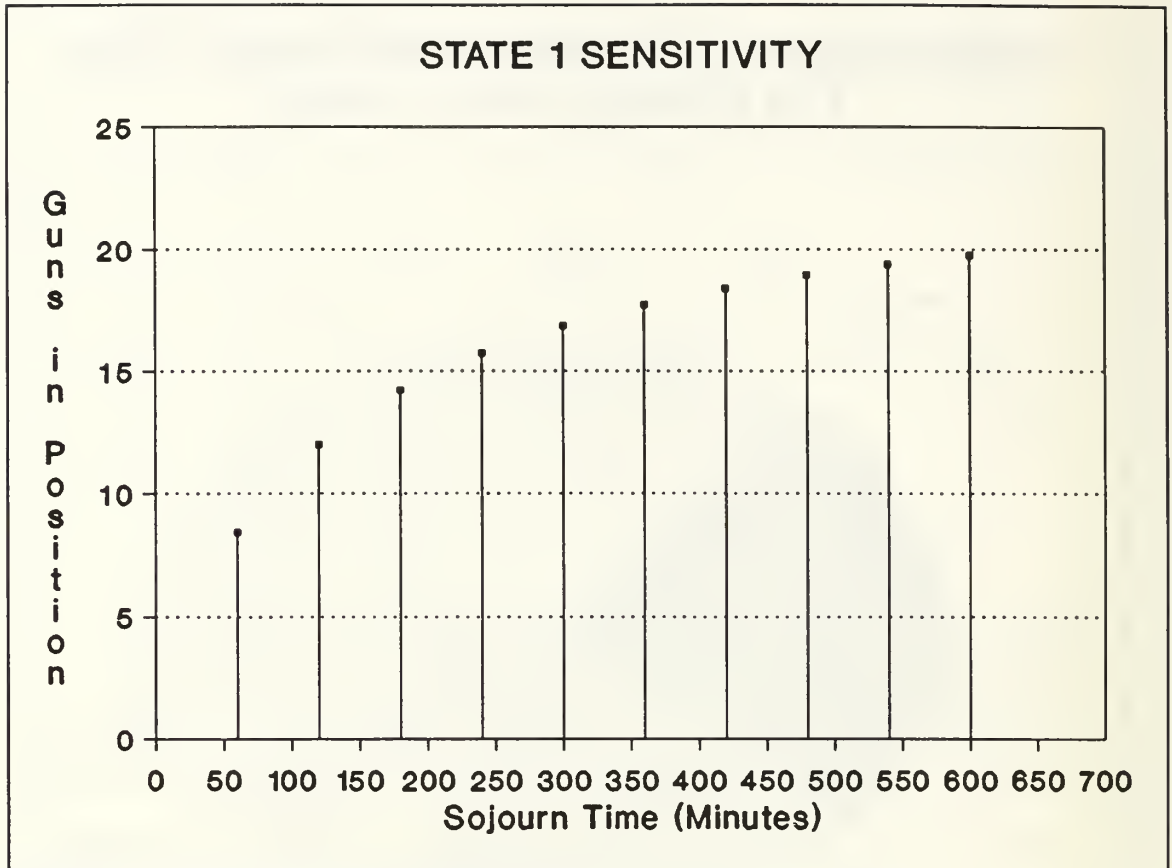
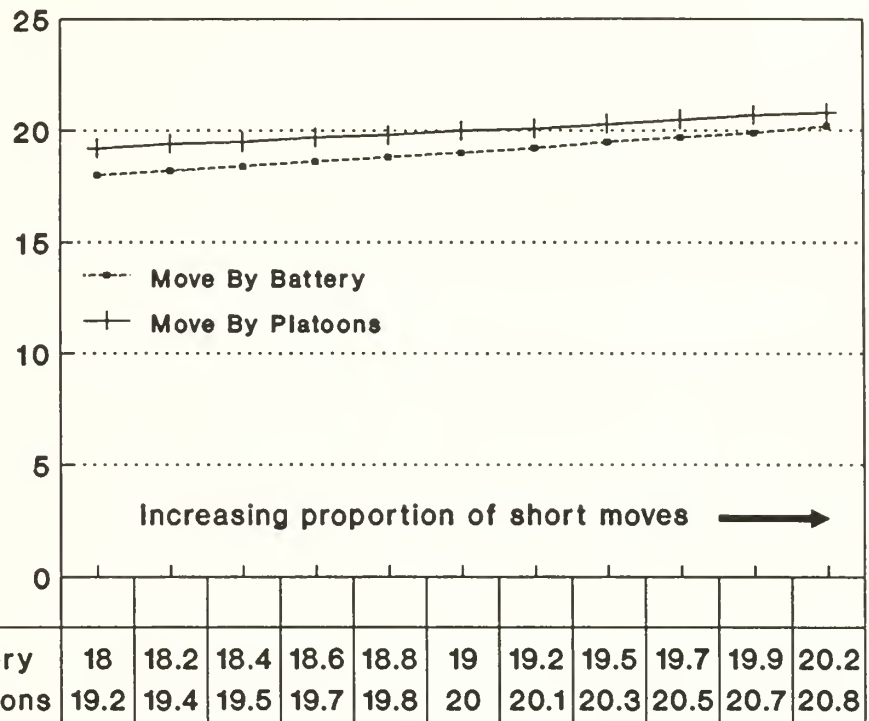


Figure 7. Sensitivity to Sojourn Time

factors that determine movement selection to get an optimum strategy.

Due to the interactive nature of the model any new scenarios can be easily added. The only requirement is to derive a transition probability matrix and mean sojourn times.

# **SENSITIVITY TO PROPORTION** **4X6 Battalion**



**Figure 8. Sensitivity to Proportion**

## **VI. SUMMARY**

### **A. CONCLUSIONS**

Based on the inputs provided to the model, the 4X6 battalion structure leads to a larger long-run expectation of guns in position. The difference between the battalion organizations varies according to the scenario. Additionally, longer sojourn times for the batteries in position with all howitzers leads to more guns up. This curve appears logarithmic in nature, however, so there is less increase in benefits at longer sojourn times. Short moves tended to result in more guns available, but the increase over long moves is slight.

Besides being used for comparison purposes, the model displays excellent properties as a planning tool. Commanders can perform sensitivity analysis on the model and see how changes in input affect the number of guns they can expect to have in position. Combined with other decision inputs such as tactical situation, counterbattery threat, and future operations, units can make better movement choices.

### **B. RECOMMENDATIONS**

It is recommended that this model be expanded to include other possible states to give better resolution. Additionally, programming the model in a language that can be run on a personal computer would allow it to be used by all artillery units. This would enable commanders to use the model in field conditions for planning purposes.

This model should be used in conjunction with other decision making tools to help select the Marine Corps artillery structure for the future. More extensive

sensitivity analysis should be performed in order to replicate the conditions Marine Corps artillery will operate under. The model itself may also be used for other comparisons including weapon systems.

### **C. SUMMARY**

The Marine Corps in the past has often relied on rather subjective analysis to select the best artillery structure. Measure of effectiveness have failed to address the basic question of artillery support: how many guns can I expect to have in position ready to fire? This model provides an answer to that question. It requires a minimum amount of input and can be easily manipulated to handle a myriad of scenarios. Increased reliance on operations research techniques will prevent a misdirection of effort. Keeping models simple will help ensure that they will be utilized by the decision makers.

To face the upcoming reductions in the military force, the Marine Corps must work smarter, not just harder. This model is a tool to allow it to do just that.

# APPENDIX A. 3X8 BATTALION STATE SPACE

TABLE 4. 3X8 BATTALION STATE SPACE

BN STATES	NUMBER OF BATTERIES IN BATTERY STATES					TOTAL GUNS AVAIL.
	1	2	3	4	5	
1	3					24
2		3				4
3			3			4
1				3		12
5					3	12
8	2	1				16
7	2		1			16
8	2			1		20
9	2				1	20
10	1	2				6
13		2	1			4
18		2		1		4
13		2			1	0
14	1		2			8
15		1	2			0
10			2	1		0
13			2		1	4
18	1			2		16
19		1		2		8
20			1	2		8
21				2	1	12

**TABLE 4. 3X8 BATTALION STATE SPACE  
(CONTINUED)**

BN STATES	NUMBER OF BATTERIES IN BATTERY STATES					TOTAL GUNS AVAIL.
	1	2	3	4	5	
<b>21</b>				2	1	12
<b>22</b>	1				2	16
<b>23</b>		1			2	8
<b>21</b>			1		2	8
<b>25</b>				1	2	12
<b>21</b>	1	1	1			8
<b>27</b>	1	1		1		12
<b>25</b>	1	1			1	12
<b>25</b>	1		1	1		12
<b>30</b>	1		1		1	12
<b>31</b>	1			1	1	16
<b>32</b>		1	1	1		4
<b>33</b>		1	1		1	4
<b>34</b>		1		1	1	8
<b>35</b>			1	1	1	8



## APPENDIX B. 4X6 BATTALION STATE SPACE

**TABLE 5. 4X6 BATTALION STATE SPACE**

BN STATE	NUMBER OF BATTERIES IN BATTERY STATES							TOTAL GUNS AVAIL.
	1	2	3	4	5	6	7	
4	4							24
8		4						0
8			4					0
4				4				16
5					1			16
5						4		6
7							4	8
8	3	4						18
8	3		4					18
19	3			4				22
21	3				1			22
12	3					4		22
19	3						1	22
14	1	3						6
16		3	1					6
16		3		4				4
17		3			1			4
18		3				4		2
19		3					1	2
10	1		3					6
21		4	3					6
22			3	1				4

TABLE 5. 4X6 BATTALION STATE SPACE (CONTINUED)

BN STATE	NUMBER OF BATTERIES IN BATTERY STATES							TOTAL GUNS AVAIL.
	1	2	3	4	5	6	7	
23			3		1			3
23			3			1		2
25			3				1	3
29	1			3				10
23		1		3				12
28			1	3				12
29				3	1			10
33				3		1		10
32				3			1	10
32	1				3			10
33		1			3			12
34			1		3			12
36				1	3			16
36					3	1		14
33					3		1	14
33	1					3		12
33		1				3		6
40			1			3		6
44				1		3		10
42					1	3		10
43						3	1	8
44	1						3	12
40		1					3	6
46			1				3	6
47				1			3	10

TABLE 5. 4X6 BATTALION STATE SPACE (CONTINUED)

BN STATE	NUMBER OF BATTERIES IN BATTERY STATES							TOTAL GUNS AVAIL.
	1	2	3	4	5	6	7	
48					1		3	16
49						1	3	8
50	2	2						12
58	2		2					12
58	2			2				20
53	2				2			20
50	2					2		16
50	2						2	16
57		2	2					0
57		2		2				8
58		2			2			8
59		2				2		4
50		2					2	4
64			2	2				8
62			2		2			8
63			2			2		4
64			2				2	4
65				2	2			16
50				2		2		12
64				2			2	12
58					2	2		12
69					2		2	12
70						2	2	8
71	2	1	1					12
72	2	1		1				16

TABLE 5. 4X6 BATTALION STATE SPACE (CONTINUED)

BN STATE	NUMBER OF BATTERIES IN BATTERY STATES							TOTAL GUNS AVAIL.
	1	2	3	4	5	6	7	
73	2	1			1			16
74	2	1				1		14
75	2	1					1	14
76	2		1	1				16
77	2		1		1			16
78	2		1			1		14
79	2		1				1	14
82	2			1	1		2	20
82	2			1		1		16
82	2			1			1	16
83	2				1	1		16
84	2				1		1	18
85	2					1	1	16
86	1	2	1					6
82	1	2		1				16
86	1	2			1			10
86	1	2				1		8
96	1	2					1	8
91		2	1	1				4
82		2	1		1			4
73		2	1			1		8
94		2	1				1	8
96		2		1	1			8
96		2		1		1		6
97		2		1			1	6



TABLE 5. 4X6 BATTALION STATE SPACE (CONTINUED)

BN STATE	NUMBER OF BATTERIES IN BATTERY STATES							TOTAL GUNS AVAIL.
	1	2	3	4	5	6	7	
98		2			1	1		6
99		2			1		1	6
100		2				1	1	4
101	1	1	2					6
102	1		2	1				14
103	1		2		1			10
104	1		2			1		6
105	1		2				1	6
106		1	2	1				4
107		1	2		1			4
108		1	2			1		2
109		1	2				1	2
110			2	1	1			6
111			2	1		1		6
112			2	1			1	6
113			2		1	1		6
114			2		1		1	6
115			2			1	1	4
116	1	1		2				14
117	1		1	2				14
118	1			2	1			18
119	1			2		1		16
120	1			2			1	16
121		1	1	2				6
122		1		2	1			12

TABLE 5. 4X6 BATTALION STATE SPACE (CONTINUED)

BN STATE	NUMBER OF BATTERIES IN BATTERY STATES							TOTAL GUNS AVAIL.
	1	2	3	4	5	6	7	
123		1		2		1		10
124		1		2			1	10
125			1	2	1			12
126			1	2		1		12
128			1	2			1	10
128				2	1	1		14
125				2	1		1	14
130				2		1	1	12
131	1	1			2			14
132	1		1		2			14
137	1			1	2			18
137	1				2	1		16
135	1				2		1	16
135		1	1		2			8
137		1		1	2			12
137		1			2	1		10
130		1			2		1	12
142			1	1	2			12
141			1		2	1		10
142			1		2		1	12
143				1	2	1		14
144				1	2		1	14
145					2	1	1	12
142	1	1				2		8
147	1		1			2		8



**TABLE 5. 4X6 BATTALION STATE SPACE (CONTINUED)**

BN STATE	NUMBER OF BATTERIES IN BATTERY STATES							TOTAL GUNS AVAIL.
	1	2	3	4	5	6	7	
148	1			1		2		12
149	1				1	2		12
150	1					2	1	10
151		1	1			2		4
162		1		1		2		8
153		1			1	2		8
154		1				2	1	6
155			1	1		2		8
156			1		1	2		8
157			1			2	1	8
158				1	1	2		12
160				1		2	1	12
160					1	2	1	12
162	1	1					2	8
162	1		1				2	8
163	1			1			2	12
164	1				1		2	12
155	1					1	2	12
160		1	1				2	8
160		1		1			2	8
160		1			1		2	8
160		1				1	2	8
170			1	1			2	8
171			1		1		2	8
172			1			1	2	6

TABLE 5. 4X6 BATTALION STATE SPACE (CONTINUED)

BN STATE	NUMBER OF BATTERIES IN BATTERY STATES							TOTAL GUNS AVAIL.
	1	2	3	4	5	6	7	
173				1	1		2	12
177				1		1	2	10
175					1	1	2	10
178	1	1	1	1				10
177	1	1	1		1			10
178	1	1	1			1		8
173	1	1	1				1	8
180		1	1	1	1			8
181		1	1	1		1		6
182		1	1	1			1	6
183			1	1	1	1		10
184			1	1	1		1	10
185				1	1	1	1	12
186	1		1	1	1			10
177	1		1	1		1		12
188	1		1	1			1	12
189	1	1		1	1			14
185	1	1		1		1		12
181	1	1		1			1	12
192	1	1			1	1		12
183	1	1			1		1	12
180	1	1				1	1	10
185	1		1		1	1		12
185	1		1		1		1	12
197	1		1			1	1	10

**TABLE 5. 4X6 BATTALION STATE SPACE (CONTINUED)**

BN STATE	NUMBER OF BATTERIES IN BATTERY STATES							TOTAL GUNS AVAIL.
	1	2	3	4	5	6	7	
198	1			1	1	1		16
199	1			1	1		1	16
209	1			1		1	1	14
201	1				1	1	1	14
202		1	1		1	1		6
207		1	1		1		1	8
204		1	1			1	1	4
209		1		1	1	1		14
208		1		1	1		1	14
207		1		1		1	1	8
208		1			1	1	1	8
209			1	1		1	1	8
210			1		1	1	1	8

## APPENDIX C. TRANSITION PROBABILITIES

All transition probabilities listed are from state 1 for both organizations.

**TABLE 6. 4X6 TRANSITION PROBABILITIES**

SCENARIO	TRANSITION PROBABILITIES TO STATE:					
	2	3	4	5	6	7
MVMNT TO CONTACT	.8	.1	.075	.025	0	0
MVMNT TO CONTACT(NITE)	.8	.1	.075	.025	0	0
EXPLOITATION (DAY)	.1	.6	.05	.25	0	0
EXPLOITATION (NITE)	.1	.6	.05	.25	0	0
DELIBERATE ATTACK	.1	.8	.025	.075	0	0
DEFENSE MBA	.05	.85	.025	.075	0	0
DELAY	.15	.80	.010	.040	0	0
REINFORCED MVMNT TO CONTACT	.9	.9	0	0	0	0
REINFORCED EXPLOITATION	.2	.8	0	0	0	0



**TABLE 7. 3X8 TRANSITION PROBABILITIES**

SCENARIO	TRANS. PROBS TO STATE:			
	2	3	4	5
MVMNT TO CONTACT	0	0	.9	.1
MVMNT TO CONTACT(NITE)	0	0	.9	.1
EXPLOITATION (DAY)	0	0	.2	.8
EXPLOITATION (NITE)	0	0	.2	.8
DELIBERATE ATTACK	0	0	.1	.9
DEFENSE MBA	0	0	.05	.95
DELAY	0	0	.15	.85
REINFORCED MVMNT TO CONTACT	.9	.1	0	0
REINFORCED EXPLOITATION	.2	.8	0	0

## APPENDIX D. MEAN SOJOURN TIMES

All sojourn times are listed in minutes.

**TABLE 8. 4X6 MEAN SOJOURN TIMES**

SCENARIO	MEAN SOJOURN TIME FOR STATE:						
	1	2	3	4	5	6	7
<b>MVMNT TO CONTACT</b>	420	145	80	130	70	130	75
<b>MVMNT TO CONTACT(NITE)</b>	630	265	145	250	130	260	140
<b>EXPLOITATION (DAY)</b>	120	145	80	130	70	135	75
<b>EXPLOITATION (NITE)</b>	60	265	145	250	130	260	140
<b>DELIBERATE ATTACK</b>	600	140	80	130	70	135	75
<b>DEFENSE MBA</b>	480	140	80	130	70	135	75
<b>DELAY</b>	120	140	80	130	70	135	75
<b>REINFORCED MVMNT TO CONTACT</b>	600	140	80	130	70	135	75
<b>REINFORCED EXPLOITATION</b>	180	140	80	130	70	135	75



**TABLE 9. 3X8 MEAN SOJOURN TIMES**

<b>SCENARIO</b>	<b>TRANS. PROBS TO STATE:</b>				
	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
<b>MVMNT TO CONTACT</b>	120	150	90	280	160
<b>MVMNT TO CONTACT(NITE)</b>	180	285	165	530	290
<b>EXPLOITATION (DAY)</b>	90	150	90	280	160
<b>EXPLOITATION (NITE)</b>	1	280	165	530	290
<b>DELIBERATE ATTACK</b>	280	150	90	280	160
<b>DEFENSE MBA</b>	480	150	90	280	160
<b>DELAY</b>	90	150	90	280	160
<b>REINFORCED MVMNT TO CONTACT</b>	360	150	90	280	160
<b>REINFORCED EXPLOITATION</b>	120	150	90	280	160

## APPENDIX E. APL FUNCTIONS

∇PMATRIX6[□]∇

∇ Z←PMATRIX6;P

- [1] ⚡THIS PROGRAM CREATES THE TRANSITION PROBABILITY MATRIX
- [2] ⚡FOR THE 4X6 BATTALION STRUCTURE. IT WILL PROMPT THE
- [3] ⚡USER FOR THE TRANSITION PROBABILITIES FOR THE VARIOUS
- [4] ⚡STATES AND THEN CREATES THE 7X7 TRANSITION MATRIX.
- [5] P← 7 7 ρ0
- [6] 'ENTER THE PROBABILITY OF TRANSITIONING FROM 6 GUNS READY TO'
- [7] '6 GUNS MOVING LONG DISTANCE'
- [8] P[1;2]←□
- [9] 'ENTER THE PROBABILITY OF TRANSITIONING FROM 6 GUNS READY TO'
- [10] '6 GUNS MOVING SHORT DISTANCE'
- [11] P[1;3]←□
- [12] 'ENTER THE PROBABILITY OF TRANSITIONING FROM 6 GUNS READY TO'
- [13] '4 GUNS READY / 2 GUNS MOVING LONG DISTANCE'
- [14] P[1;4]←□
- [15] 'ENTER THE PROBABILITY OF TRANSITIONING FROM 6 GUNS READY TO'
- [16] '4 GUNS READY / 2 GUNS MOVING SHORT DISTANCE'
- [17] P[1;5]←□
- [18] P[2;1]←1
- [19] P[3;1]←1
- [20] P[4;6]←1
- [21] P[5;7]←1
- [22] P[6;1]←1
- [23] P[7;1]←1
- [24] Z←P

∇

∇PMATRIX8[□]∇

∇ Z←PMATRIX8;P

- [1] ⚡THIS PROGRAM CREATES THE TRANSITION PROBABILITY MATRIX
- [2] ⚡FOR THE 3X8 BATTALION STRUCTURE. IT WILL PROMPT THE
- [3] ⚡USER FOR THE TRANSITION PROBABILITIES FOR THE VARIOUS
- [4] ⚡STATES AND THEN CREATES THE 5X5 TRANSITION MATRIX.
- [5] P← 5 5 ρ0
- [6] 'ENTER THE PROBABILITY OF TRANSITIONING FROM 8 GUNS READY TO'
- [7] '8 GUNS MOVING LONG DISTANCE'
- [8] P[1;2]←□
- [9] 'ENTER THE PROBABILITY OF TRANSITIONING FROM 8 GUNS READY TO'
- [10] '8 GUNS MOVING SHORT DISTANCE'

[11] P[1;3]-□  
 [12] 'ENTER THE PROBABILITY OF TRANSITIONING FROM 8 GUNS READY TO'  
 [13] '4 GUNS READY / 4 GUNS MOVING LONG DISTANCE'  
 [14] P[1;4]-□  
 [15] 'ENTER THE PROBABILITY OF TRANSITIONING FROM 8 GUNS READY TO'  
 [16] '4 GUNS READY / 4 GUNS MOVING SHORT DISTANCE'  
 [17] P[1;5]-□  
 [18] P[2;1]-1  
 [19] P[3;1]-1  
 [20] P[4;1]-0.5  
 [21] P[4;4]-0.5  
 [22] P[5;1]-0.5  
 [23] P[5;5]-0.5  
 [24] Z-P  
 ▽

▽SMATRIX6[□]▽

▽ Z-SMATRIX6;S

[1] ρTHIS PROGRAM CREATES THE MATRIX OF MEAN SOJOURN TIMES FOR  
 [2] ρTHE 4X6 BATTALION STRUCTURE. IT WILL PROMPT THE USER FOR  
 [3] ρMEAN SOJOURN TIMES FOR THE VARIOUS STATES.  
 [4] S- 1 7 ρ0  
 [5] 'ENTER ALL TIMES IN MINUTES'  
 [6] 'ENTER THE MEAN SOJOURN TIME FOR THE BATTERY TO BE IN  
 POSITION'  
 [7] 'WITH 6 GUNS READY'  
 [8] S[1;1]-□  
 [9] 'ENTER THE MEAN SOJOURN TIME FOR THE BATTERY TO BE MOVING  
 A'  
 [10] 'LONG DISTANCE WITH 6 GUNS'  
 [11] S[1;2]-□  
 [12] 'ENTER THE MEAN SOJOURN TIME FOR THE BATTERY TO BE MOVING  
 A'  
 [13] 'SHORT DISTANCE WITH 6 GUNS'  
 [14] S[1;3]-□  
 [15] 'ENTER THE MEAN SOJOURN TIME FOR THE BATTERY TO BE MOVING  
 A'  
 [16] 'LONG DISTANCE WITH 2 GUNS AND LEAVING 4 GUNS IN POSITION'  
 [17] S[1;4]-□  
 [18] 'ENTER THE MEAN SOJOURN TIME FOR THE BATTERY TO BE MOVING  
 A'  
 [19] 'SHORT DISTANCE WITH 2 GUNS AND LEAVING 4 GUNS IN POSITION'  
 [20] S[1;5]-□  
 [21] 'ENTER THE MEAN SOJOURN TIME FOR THE BATTERY TO BE MOVING  
 A'

[22] 'LONG DISTANCE WITH 4 GUNS AND LEAVING 2 GUNS IN POSITION'  
 [23] S[1;6]-□  
 [24] 'ENTER THE MEAN SOJOURN TIME FOR THE BATTERY TO BE MOVING  
 A'  
 [25] 'SHORT DISTANCE WITH 4 GUNS AND LEAVING 2 GUNS IN POSITION'  
 [26] S[1;7]-□  
 [27] Z-S

▽

▽SMATRIX8[□]▽

▽Z-SMATRIX8;S

[1] ⚙THIS PROGRAM CREATES THE MATRIX OF MEAN SOJOURN TIMES FOR  
 [2] ⚙THE 3X8 BATTALION STRUCTURE. IT WILL PROMPT THE USER FOR  
 [3] ⚙MEAN SOJOURN TIMES FOR THE VARIOUS STATES.  
 [4] S- 1 5 ρ0  
 [5] 'ENTER ALL TIMES IN MINUTES'  
 [6] 'ENTER THE MEAN SOJOURN TIME FOR THE BATTERY TO BE IN  
 POSITION'  
 [7] 'WITH 8 GUNS READY'  
 [8] S[1;1]-□  
 [9] 'ENTER THE MEAN SOJOURN TIME FOR THE BATTERY TO BE MOVING  
 A'  
 [10] 'LONG DISTANCE WITH 8 GUNS'  
 [11] S[1;2]-□  
 [12] 'ENTER THE MEAN SOJOURN TIME FOR THE BATTERY TO BE MOVING  
 A'  
 [13] 'SHORT DISTANCE WITH 8 GUNS'  
 [14] S[1;3]-□  
 [15] 'ENTER THE MEAN SOJOURN TIME FOR THE BATTERY TO BE MOVING  
 A'  
 [16] 'LONG DISTANCE WITH 4 GUNS AND LEAVING 4 GUNS IN POSITION'  
 [17] S[1;4]-□  
 [18] 'ENTER THE MEAN SOJOURN TIME FOR THE BATTERY TO BE MOVING  
 A'  
 [19] 'SHORT DISTANCE WITH 4 GUNS AND LEAVING 4 GUNS IN POSITION'  
 [20] S[1;5]-□  
 [21] Z-S

▽

▽INFGEN[□]▽

▽Z-S INFGEN P;I;J;A;INV

[1] ⚙THIS PROGRAM CREATES THE INFINITESIMAL GENERATOR  
 [2] A-Px0

```

[3] INV←+S
[4] I←0
[5] I←I+1
[6] J←0
[7] J←J+1
[8] A[I;J]←INV[1;I]xP[I;J]
[9] A[I;I]←-(INV[1;I])
[10] →7+((J=ρP[1;])x4)
[11] →5+((I=ρP[1;])x7)
[12] Z←A
    ▽

```

```

    ▽COMPUTE[□]▽
    ▽ Z←S COMPUTE P;I;PI;TMAT
[1] ⌘THIS PROGRAM COMPUTES THE MEAN RECURRENT TIMES FOR
[2] ⌘EACH STATE SPACE.
[3] PI←S PIMAT P
[4] TMAT←Sx0
[5] I←0
[6] I←I+1
[7] TMAT[1;I]←S[1;I]+PI[I]
[8] →6+((I=ρPI)x3)
[9] Z←TMAT
    ▽

```

```

    ▽PIMAT[□]▽
    ▽ Z←S PIMAT P;N;M;AMAT;ZEMAT
[1] ⌘THIS PROGRAM COMPUTES THE LONG-RUN EQUILIBRIUM
[2] ⌘PROBABILITIES FOR EACH STATE.
[3] N←S
[4] M←P
[5] ZEMAT←M[1;]x0
[6] ZEMAT←ZEMAT,1
[7] AMAT←Q(AMAT←((N INFGEN M),1))
[8] Z←ZEMAT⊖AMAT
    ▽

```

```

    ▽DATA[□]▽
    ▽ DATA;X;Y;G
[1] ⌘THIS PROGRAM USES THE PROGRAMS COMPUTE, INFGEN, PIMAT,
[2] ⌘PMATRIX6, PMATRIX8, SMATRIX6, AND SMATRIX8 TO DISPLAY THE

```



[3] ♡LONG-RUN EQUILIBRIUM PROBABILITIES AND MEAN RECURRENCE  
 TIMES  
 [4] ♡FOR EACH BATTERY CONFIGURATION.  
 [5] 'IF YOU ARE GENERATING DATA FOR 6 GUN BATTERY, ENTER A 6'  
 [6] 'IF YOU ARE GENERATING DATA FOR 8 GUN BATTERY, ENTER A 8'  
 [7] G-□  
 [8] -9+((6=G)x1)  
 [9] -24-((8=G)x7)  
 [10] X-SMATRIX6  
 [11] Y-PMATRIX6  
 [12] 'LONG-RUN EQUILIBRIUM PROBABILITIES'  
 [13] □-X PIMAT Y  
 [14] 'MEAN RECURRENCE TIMES'  
 [15] □-X COMPUTE Y  
 [16] -26  
 [17] X-SMATRIX8  
 [18] Y-PMATRIX8  
 [19] 'LONG-RUN EQUILIBRIUM PROBABILITIES'  
 [20] □-X PIMAT Y  
 [21] 'MEAN RECURRENCE TIMES'  
 [22] □-X COMPUTE Y  
 [23] -26  
 [24] 'INVALID NUMBER. TRY AGAIN.'  
 [25] -5  
 ▽

▽BNPIE6[□]▽

▽BNPIE6;B;C;D;X;Y

[1] ♡THIS PROGRAM COMPUTES THE LONG-RUN EQUILIBRIUM  
 PROBABILITIES  
 [2] ♡FOR THE 4X6 ARTILLERY BATTALION BY COMBINING THE LONG-RUN  
 [3] ♡EQUILIBRIUM PROBABILITIES FOR THE INDIVIDUAL BATTERIES.  
 [4] B- 21 10 ρ0  
 [5] X-SMATRIX6  
 [6] Y-PMATRIX6  
 [7] C-X PIMAT Y  
 [8] B[1;1]-C[1]\*4  
 [9] B[1;2]-C[2]\*4  
 [10] B[1;3]-C[3]\*4  
 [11] B[1;4]-C[4]\*4  
 [12] B[1;5]-C[5]\*4  
 [13] B[1;6]-C[6]\*4  
 [14] B[1;7]-C[7]\*4  
 [15] B[1;8]-(C[1]\*3)xC[2]x4  
 [16] B[1;9]-(C[1]\*3)xC[3]x4



[17]  $B[1;10]-(C[1]^3)xC[4]x4$   
 [18]  $B[2;1]-(C[1]^3)xC[5]x4$   
 [19]  $B[2;2]-(C[1]^3)xC[6]x4$   
 [20]  $B[2;3]-(C[1]^3)xC[7]x4$   
 [21]  $B[2;4]-(C[2]^3)xC[1]x4$   
 [22]  $B[2;5]-(C[2]^3)xC[3]x4$   
 [23]  $B[2;6]-(C[2]^3)xC[4]x4$   
 [24]  $B[2;7]-(C[2]^3)xC[5]x4$   
 [25]  $B[2;8]-(C[2]^3)xC[6]x4$   
 [26]  $B[2;9]-(C[2]^3)xC[7]x4$   
 [27]  $B[2;10]-(C[3]^3)xC[1]x4$   
 [28]  $B[3;1]-(C[3]^3)xC[2]x4$   
 [29]  $B[3;2]-(C[3]^3)xC[4]x4$   
 [30]  $B[3;3]-(C[3]^3)xC[5]x4$   
 [31]  $B[3;4]-(C[3]^3)xC[6]x4$   
 [32]  $B[3;5]-(C[3]^3)xC[7]x4$   
 [33]  $B[3;6]-(C[4]^3)xC[1]x4$   
 [34]  $B[3;7]-(C[4]^3)xC[2]x4$   
 [35]  $B[3;8]-(C[4]^3)xC[3]x4$   
 [36]  $B[3;9]-(C[4]^3)xC[5]x4$   
 [37]  $B[3;10]-(C[4]^3)xC[6]x4$   
 [38]  $B[4;1]-(C[4]^3)xC[7]x4$   
 [39]  $B[4;2]-(C[5]^3)xC[1]x4$   
 [40]  $B[4;3]-(C[5]^3)xC[2]x4$   
 [41]  $B[4;4]-(C[5]^3)xC[3]x4$   
 [42]  $B[4;5]-(C[5]^3)xC[4]x4$   
 [43]  $B[4;6]-(C[5]^3)xC[6]x4$   
 [44]  $B[4;7]-(C[5]^3)xC[7]x4$   
 [45]  $B[4;8]-(C[6]^3)xC[1]x4$   
 [46]  $B[4;9]-(C[6]^3)xC[2]x4$   
 [47]  $B[4;10]-(C[6]^3)xC[3]x4$   
 [48]  $B[5;1]-(C[6]^3)xC[4]x4$   
 [49]  $B[5;2]-(C[6]^3)xC[5]x4$   
 [50]  $B[5;3]-(C[6]^3)xC[7]x4$   
 [51]  $B[5;4]-(C[7]^3)xC[1]x4$   
 [52]  $B[5;5]-(C[7]^3)xC[2]x4$   
 [53]  $B[5;6]-(C[7]^3)xC[3]x4$   
 [54]  $B[5;7]-(C[7]^3)xC[4]x4$   
 [55]  $B[5;8]-(C[7]^3)xC[5]x4$   
 [56]  $B[5;9]-(C[7]^3)xC[6]x4$   
 [57]  $B[5;10]-(C[1]^2)x(C[2]^2)x6$   
 [58]  $B[6;1]-(C[1]^2)x(C[3]^2)x6$   
 [59]  $B[6;2]-(C[1]^2)x(C[4]^2)x6$   
 [60]  $B[6;3]-(C[1]^2)x(C[5]^2)x6$   
 [61]  $B[6;4]-(C[1]^2)x(C[6]^2)x6$   
 [62]  $B[6;5]-(C[1]^2)x(C[7]^2)x6$   
 [63]  $B[6;6]-(C[2]^2)x(C[3]^2)x6$

[64]  $B[6;7]-(C[2]^2)x(C[4]^2)x6$   
 [65]  $B[6;8]-(C[2]^2)x(C[5]^2)x6$   
 [66]  $B[6;9]-(C[2]^2)x(C[6]^2)x6$   
 [67]  $B[6;1]-(C[2]^2)x(C[7]^2)x6$   
 [68]  $B[7;1]-(C[3]^2)x(C[4]^2)x6$   
 [69]  $B[7;2]-(C[3]^2)x(C[5]^2)x6$   
 [70]  $B[7;3]-(C[3]^2)x(C[6]^2)x6$   
 [71]  $B[7;4]-(C[3]^2)x(C[7]^2)x6$   
 [72]  $B[7;5]-(C[4]^2)x(C[5]^2)x6$   
 [73]  $B[7;6]-(C[4]^2)x(C[6]^2)x6$   
 [74]  $B[7;7]-(C[4]^2)x(C[7]^2)x6$   
 [75]  $B[7;8]-(C[5]^2)x(C[6]^2)x6$   
 [76]  $B[7;9]-(C[5]^2)x(C[7]^2)x6$   
 [77]  $B[7;10]-(C[6]^2)x(C[7]^2)x6$   
 [78]  $B[8;1]-(C[1]^2)x C[2]x C[3]x12$   
 [79]  $B[8;2]-(C[1]^2)x C[2]x C[4]x12$   
 [80]  $B[8;3]-(C[1]^2)x C[2]x C[5]x12$   
 [81]  $B[8;4]-(C[1]^2)x C[2]x C[6]x12$   
 [82]  $B[8;5]-(C[1]^2)x C[2]x C[7]x12$   
 [83]  $B[8;6]-(C[1]^2)x C[3]x C[4]x12$   
 [84]  $B[8;7]-(C[1]^2)x C[3]x C[5]x12$   
 [85]  $B[8;8]-(C[1]^2)x C[3]x C[6]x12$   
 [86]  $B[8;9]-(C[1]^2)x C[3]x C[7]x12$   
 [87]  $B[8;10]-(C[1]^2)x C[4]x C[5]x12$   
 [88]  $B[9;1]-(C[1]^2)x C[4]x C[6]x12$   
 [89]  $B[9;2]-(C[1]^2)x C[4]x C[7]x12$   
 [90]  $B[9;3]-(C[1]^2)x C[5]x C[6]x12$   
 [91]  $B[9;4]-(C[1]^2)x C[5]x C[7]x12$   
 [92]  $B[9;5]-(C[1]^2)x C[6]x C[7]x12$   
 [93]  $B[9;6]-(C[2]^2)x C[1]x C[3]x12$   
 [94]  $B[9;7]-(C[2]^2)x C[1]x C[4]x12$   
 [95]  $B[9;8]-(C[2]^2)x C[1]x C[5]x12$   
 [96]  $B[9;9]-(C[2]^2)x C[1]x C[6]x12$   
 [97]  $B[9;10]-(C[2]^2)x C[1]x C[7]x12$   
 [98]  $B[10;1]-(C[2]^2)x C[3]x C[4]x12$   
 [99]  $B[10;2]-(C[2]^2)x C[3]x C[5]x12$   
 [100]  $B[10;3]-(C[2]^2)x C[3]x C[6]x12$   
 [101]  $B[10;4]-(C[2]^2)x C[3]x C[7]x12$   
 [102]  $B[10;5]-(C[2]^2)x C[4]x C[5]x12$   
 [103]  $B[10;6]-(C[2]^2)x C[4]x C[6]x12$   
 [104]  $B[10;7]-(C[2]^2)x C[4]x C[7]x12$   
 [105]  $B[10;8]-(C[2]^2)x C[5]x C[6]x12$   
 [106]  $B[10;9]-(C[2]^2)x C[5]x C[7]x12$   
 [107]  $B[10;10]-(C[2]^2)x C[6]x C[7]x12$   
 [108]  $B[11;1]-(C[3]^2)x C[1]x C[2]x12$   
 [109]  $B[11;2]-(C[3]^2)x C[1]x C[4]x12$   
 [110]  $B[11;3]-(C[3]^2)x C[1]x C[5]x12$

[111]  $B[11;4] \sim (C[3]^2)xC[1]xC[6]x12$   
 [112]  $B[11;5] \sim (C[3]^2)xC[1]xC[7]x12$   
 [113]  $B[11;6] \sim (C[3]^2)xC[2]xC[4]x12$   
 [114]  $B[11;7] \sim (C[3]^2)xC[2]xC[5]x12$   
 [115]  $B[11;8] \sim (C[3]^2)xC[2]xC[6]x12$   
 [116]  $B[11;9] \sim (C[3]^2)xC[2]xC[7]x12$   
 [117]  $B[11;10] \sim (C[3]^2)xC[4]xC[5]x12$   
 [118]  $B[12;1] \sim (C[3]^2)xC[4]xC[6]x12$   
 [119]  $B[12;2] \sim (C[3]^2)xC[4]xC[7]x12$   
 [120]  $B[12;3] \sim (C[3]^2)xC[5]xC[6]x12$   
 [121]  $B[12;4] \sim (C[3]^2)xC[5]xC[7]x12$   
 [122]  $B[12;5] \sim (C[3]^2)xC[6]xC[7]x12$   
 [123]  $B[12;6] \sim (C[4]^2)xC[1]xC[2]x12$   
 [124]  $B[12;7] \sim (C[4]^2)xC[1]xC[3]x12$   
 [125]  $B[12;8] \sim (C[4]^2)xC[1]xC[5]x12$   
 [126]  $B[12;9] \sim (C[4]^2)xC[1]xC[6]x12$   
 [127]  $B[12;10] \sim (C[4]^2)xC[1]xC[7]x12$   
 [128]  $B[13;1] \sim (C[4]^2)xC[2]xC[3]x12$   
 [129]  $B[13;2] \sim (C[4]^2)xC[2]xC[5]x12$   
 [130]  $B[13;3] \sim (C[4]^2)xC[2]xC[6]x12$   
 [131]  $B[13;4] \sim (C[4]^2)xC[2]xC[7]x12$   
 [132]  $B[13;5] \sim (C[4]^2)xC[3]xC[5]x12$   
 [133]  $B[13;6] \sim (C[4]^2)xC[3]xC[6]x12$   
 [134]  $B[13;7] \sim (C[4]^2)xC[3]xC[7]x12$   
 [135]  $B[13;8] \sim (C[4]^2)xC[5]xC[6]x12$   
 [136]  $B[13;9] \sim (C[4]^2)xC[5]xC[7]x12$   
 [137]  $B[13;10] \sim (C[4]^2)xC[6]xC[7]x12$   
 [138]  $B[14;1] \sim (C[5]^2)xC[1]xC[2]x12$   
 [139]  $B[14;2] \sim (C[5]^2)xC[1]xC[3]x12$   
 [140]  $B[14;3] \sim (C[5]^2)xC[1]xC[4]x12$   
 [141]  $B[14;4] \sim (C[5]^2)xC[1]xC[6]x12$   
 [142]  $B[14;5] \sim (C[5]^2)xC[1]xC[7]x12$   
 [143]  $B[14;6] \sim (C[5]^2)xC[2]xC[3]x12$   
 [144]  $B[14;7] \sim (C[5]^2)xC[2]xC[4]x12$   
 [145]  $B[14;8] \sim (C[5]^2)xC[2]xC[6]x12$   
 [146]  $B[14;9] \sim (C[5]^2)xC[2]xC[7]x12$   
 [147]  $B[14;10] \sim (C[5]^2)xC[3]xC[4]x12$   
 [148]  $B[15;1] \sim (C[5]^2)xC[3]xC[6]x12$   
 [149]  $B[15;2] \sim (C[5]^2)xC[3]xC[7]x12$   
 [150]  $B[15;3] \sim (C[5]^2)xC[4]xC[6]x12$   
 [151]  $B[15;4] \sim (C[5]^2)xC[4]xC[7]x12$   
 [152]  $B[15;5] \sim (C[5]^2)xC[6]xC[7]x12$   
 [153]  $B[15;6] \sim (C[6]^2)xC[1]xC[2]x12$   
 [154]  $B[15;7] \sim (C[6]^2)xC[1]xC[3]x12$   
 [155]  $B[15;8] \sim (C[6]^2)xC[1]xC[4]x12$   
 [156]  $B[15;9] \sim (C[6]^2)xC[1]xC[5]x12$   
 [157]  $B[15;10] \sim (C[6]^2)xC[1]xC[7]x12$

[158]  $B[16;1] - (C[6]^*2)xC[2]xC[3]x12$   
 [159]  $B[16;2] - (C[6]^*2)xC[2]xC[4]x12$   
 [160]  $B[16;3] - (C[6]^*2)xC[2]xC[5]x12$   
 [161]  $B[16;4] - (C[6]^*2)xC[2]xC[7]x12$   
 [162]  $B[16;5] - (C[6]^*2)xC[3]xC[4]x12$   
 [163]  $B[16;6] - (C[6]^*2)xC[3]xC[5]x12$   
 [164]  $B[16;7] - (C[6]^*2)xC[3]xC[7]x12$   
 [165]  $B[16;8] - (C[6]^*2)xC[4]xC[5]x12$   
 [166]  $B[16;9] - (C[6]^*2)xC[4]xC[7]x12$   
 [167]  $B[16;10] - (C[6]^*2)xC[5]xC[7]x12$   
 [168]  $B[17;1] - (C[7]^*2)xC[1]xC[2]x12$   
 [169]  $B[17;2] - (C[7]^*2)xC[1]xC[3]x12$   
 [170]  $B[17;3] - (C[7]^*2)xC[1]xC[4]x12$   
 [171]  $B[17;4] - (C[7]^*2)xC[1]xC[5]x12$   
 [172]  $B[17;5] - (C[7]^*2)xC[1]xC[6]x12$   
 [173]  $B[17;6] - (C[7]^*2)xC[2]xC[3]x12$   
 [174]  $B[17;7] - (C[7]^*2)xC[2]xC[4]x12$   
 [175]  $B[17;8] - (C[7]^*2)xC[2]xC[5]x12$   
 [176]  $B[17;9] - (C[7]^*2)xC[2]xC[6]x12$   
 [177]  $B[17;10] - (C[7]^*2)xC[3]xC[4]x12$   
 [178]  $B[18;1] - (C[7]^*2)xC[3]xC[5]x12$   
 [179]  $B[18;2] - (C[7]^*2)xC[3]xC[6]x12$   
 [180]  $B[18;3] - (C[7]^*2)xC[4]xC[5]x12$   
 [181]  $B[18;4] - (C[7]^*2)xC[4]xC[6]x12$   
 [182]  $B[18;5] - (C[7]^*2)xC[5]xC[6]x12$   
 [183]  $B[18;6] - C[1]xC[2]xC[3]xC[4]x24$   
 [184]  $B[18;7] - C[1]xC[2]xC[3]xC[5]x24$   
 [185]  $B[18;8] - C[1]xC[2]xC[3]xC[6]x24$   
 [186]  $B[18;9] - C[1]xC[2]xC[3]xC[7]x24$   
 [187]  $B[18;10] - C[2]xC[3]xC[4]xC[5]x24$   
 [188]  $B[19;1] - C[2]xC[3]xC[4]xC[6]x24$   
 [189]  $B[19;2] - C[2]xC[3]xC[4]xC[7]x24$   
 [190]  $B[19;3] - C[3]xC[4]xC[5]xC[6]x24$   
 [191]  $B[19;4] - C[3]xC[4]xC[5]xC[7]x24$   
 [192]  $B[19;5] - C[4]xC[5]xC[6]xC[7]x24$   
 [193]  $B[19;6] - C[1]xC[3]xC[4]xC[5]x24$   
 [194]  $B[19;7] - C[1]xC[3]xC[4]xC[6]x24$   
 [195]  $B[19;8] - C[1]xC[3]xC[4]xC[7]x24$   
 [196]  $B[19;9] - C[1]xC[2]xC[4]xC[5]x24$   
 [197]  $B[19;10] - C[1]xC[2]xC[4]xC[6]x24$   
 [198]  $B[20;1] - C[1]xC[2]xC[4]xC[7]x24$   
 [199]  $B[20;2] - C[1]xC[2]xC[5]xC[6]x24$   
 [200]  $B[20;3] - C[1]xC[2]xC[5]xC[7]x24$   
 [201]  $B[20;4] - C[1]xC[2]xC[6]xC[7]x24$   
 [202]  $B[20;5] - C[1]xC[3]xC[5]xC[6]x24$   
 [203]  $B[20;6] - C[1]xC[3]xC[5]xC[7]x24$   
 [204]  $B[20;7] - C[1]xC[3]xC[6]xC[7]x24$

- [205]  $B[20;8] - C[1] \times C[4] \times C[5] \times C[6] \times 24$   
 [206]  $B[20;9] - C[1] \times C[4] \times C[5] \times C[7] \times 24$   
 [207]  $B[20;10] - C[1] \times C[4] \times C[6] \times C[7] \times 24$   
 [208]  $B[21;1] - C[1] \times C[5] \times C[6] \times C[7] \times 24$   
 [209]  $B[21;2] - C[2] \times C[3] \times C[5] \times C[6] \times 24$   
 [210]  $B[21;3] - C[2] \times C[3] \times C[5] \times C[7] \times 24$   
 [211]  $B[21;4] - C[2] \times C[3] \times C[6] \times C[7] \times 24$   
 [212]  $B[21;5] - C[2] \times C[4] \times C[5] \times C[6] \times 24$   
 [213]  $B[21;6] - C[2] \times C[4] \times C[5] \times C[7] \times 24$   
 [214]  $B[21;7] - C[2] \times C[4] \times C[6] \times C[7] \times 24$   
 [215]  $B[21;8] - C[2] \times C[5] \times C[6] \times C[7] \times 24$   
 [216]  $B[21;9] - C[3] \times C[4] \times C[6] \times C[7] \times 24$   
 [217]  $B[21;10] - C[3] \times C[5] \times C[6] \times C[7] \times 24$   
 [218] 'BN LONG-RUN EQUILIBRIUM PROBABILITIES'  
 [219]  $\square - B$   
 [220]  $D - 1 \ 13 \ \rho_0$   
 [221]  $D[1;1] - B[1;2] + B[1;3] + B[2;5] + B[3;1] + B[6;6]$   
 [222]  $D[1;2] - B[2;8] + B[2;9] + B[3;4] + B[3;5] + B[10;3] + B[10;4] + B[11;8]$   
            $+ B[11;9]$   
  
 [223]  $D[1;3] - B[2;6] + B[2;7] + B[3;2] + B[3;3] + B[6;9] + B[6;10] + B[7;4]$   
            $+ B[10;1] + B[10;2] + B[10;10] + B[11;6] + B[11;7] + B[12;5]$   
            $+ B[16;1] + B[17;6] + B[21;4]$   
 [224]  $D[1;4] - B[2;4] + B[2;10] + B[4;9] + B[4;10] + B[5;5] + B[5;6] + B[9;6]$   
            $+ B[10;6] + B[10;7] + B[10;8] + B[10;9] + B[11;1] + B[12;1]$   
            $+ B[12;2] + B[12;3] + B[12;4] + B[16;4] + B[16;7] + B[17;9]$   
            $+ B[18;2] + B[19;1] + B[19;2] + B[21;2] + B[21;3]$   
 [225]  $D[1;5] - B[1;7] + B[5;3] + B[6;7] + B[6;8] + B[7;1] + B[7;2] + B[7;10]$   
            $+ B[9;9] + B[9;10] + B[10;5] + B[11;4] + B[11;5] + B[11;10]$   
            $+ B[13;1] + B[14;6] + B[15;6] + B[15;7] + B[16;2] + B[16;3]$   
            $+ B[16;5] + B[17;1] + B[17;2] + B[17;7] + B[17;8] + B[17;10]$   
            $+ B[18;1] + B[18;8] + B[18;9] + B[18;10] + B[21;7] + B[21;8]$   
            $+ B[21;9] + B[21;10] + B[1;6] + B[16;6] + B[5;9]$   
 [226]  $D[1;6] - B[5;1] + B[5;2] + B[5;7] + B[5;8] + B[9;7] + B[9;8] + B[11;2]$   
            $+ B[11;3] + B[13;3] + B[13;4] + B[13;6] + B[13;7] + B[14;8]$   
            $+ B[14;9] + B[15;1] + B[15;2] + B[15;10] + B[16;9] + B[16;10]$   
            $+ B[17;5] + B[18;4] + B[18;5] + B[18;6] + B[18;7] + B[19;3]$   
            $+ B[19;4] + B[20;4] + B[20;7] + B[21;5] + B[21;6]$   
 [227]  $D[1;7] - B[3;7] + B[3;8] + B[4;3] + B[4;4] + B[4;8] + B[5;4] + B[5;10]$   
            $+ B[6;1] + B[7;6] + B[7;7] + B[7;8] + B[7;9] + B[8;1] + B[13;2]$   
            $+ B[13;5] + B[13;10] + B[14;7] + B[14;10] + B[15;5] + B[15;8]$   
            $+ B[15;9] + B[16;8] + B[17;3] + B[17;4] + B[18;3] + B[19;5]$   
            $+ B[19;7] + B[19;8] + B[19;10] + B[20;1] + B[20;2] + B[20;3]$   
            $+ B[20;5] + B[20;6]$   
 [228]  $D[1;8] - B[3;10] + B[4;1] + B[4;6] + B[4;7] + B[8;4] + B[8;5] + B[8;8]$   
            $+ B[8;9] + B[12;6] + B[12;7] + B[13;8] + B[13;9] + B[14;1]$



+ B[14;2] + B[15;3] + B[15;4] + B[19;6] + B[19;9] + B[20;10]  
 + B[21;1]  
 [229] D[1;9]-B[1;4] + B[1;5] + B[3;9] + B[4;5] + B[6;4] + B[6;5] + B[7;5]  
 + B[8;2] + B[8;3] + B[8;6] + B[8;7] + B[9;5] + B[12;9] + B[12;10]  
 + B[14;4] + B[14;5] + B[20;8] + B[20;9]  
 [230] D[1;10]-B[1;8] + B[1;9] + B[3;6] + B[4;2] + B[9;1] + B[9;2] + B[9;3]  
 + B[9;4] + B[12;8] + B[14;3]  
 [231] D[1;11]-B[2;2] + B[2;3] + B[6;2] + B[6;3] + B[8;10]  
 [232] D[1;12]-B[1;10] + B[2;1]  
 [233] D[1;13]-B[1;1]  
 [234] 'LONG-RUN EQUILIBRIUM PROBABILITIES FOR STATES WITH TOTALS'  
 [235] 'OF 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24 GUNS IN POSITION'  
 [236] □-D  
 [237] E-(D[1;2]x2) + (D[1;3]x4) + (D[1;4]x6) + (D[1;5]x8) + (D[1;6]x10)  
 + (D[1;7]x12) + (D[1;8]x14) + (D[1;9]x16) + (D[1;10]x18)  
 + (D[1;11]x20) + (D[1;12]x22) + (D[1;13]x24)  
 [238] 'LONG-RUN EXPECTATION OF GUNS IN POSITION'  
 [239] □-E  
 ▽

▽BNPIE8[□]▽

▽ BNPIE8;B;C;D;X;Y

[1] ρTHIS PROGRAM COMPUTES THE LONG-RUN EQUILIBRIUM  
 PROBABILITIES  
 [2] ρFOR THE 3X8 ARTILLERY BATTALION BY COMBINING THE LONG-RUN  
 [3] ρEQUILIBRIUM PROBABILITIES FOR THE INDIVIDUAL BATTERIES.  
 [4] B~ 7 5 ρ0  
 [5] X~SMATRIX8  
 [6] Y~PMATRIX8  
 [7] C~X PIMAT Y  
 [8] B[1;1]~C[1]\*3  
 [9] B[1;2]~C[2]\*3  
 [10] B[1;3]~C[3]\*3  
 [11] B[1;4]~C[4]\*3  
 [12] B[1;5]~C[5]\*3  
 [13] B[2;1]~(C[1]\*2)x C[2]x3  
 [14] B[2;2]~(C[1]\*2)x C[3]x3  
 [15] B[2;3]~(C[1]\*2)x C[4]x3  
 [16] B[2;4]~(C[1]\*2)x C[5]x3  
 [17] B[2;5]~(C[2]\*2)x C[1]x3  
 [18] B[3;1]~(C[2]\*2)x C[3]x3  
 [19] B[3;2]~(C[2]\*2)x C[4]x3  
 [20] B[3;3]~(C[2]\*2)x C[5]x3  
 [21] B[3;4]~(C[3]\*2)x C[1]x3  
 [22] B[3;5]~(C[3]\*2)x C[2]x3



- [23]  $B[4;1] - (C[3]^2) \times C[4] \times 3$   
 [24]  $B[4;2] - (C[3]^2) \times C[5] \times 3$   
 [25]  $B[4;3] - (C[4]^2) \times C[1] \times 3$   
 [26]  $B[4;4] - (C[4]^2) \times C[2] \times 3$   
 [27]  $B[4;5] - (C[4]^2) \times C[3] \times 3$   
 [28]  $B[5;1] - (C[4]^2) \times C[5] \times 3$   
 [29]  $B[5;2] - (C[5]^2) \times C[1] \times 3$   
 [30]  $B[5;3] - (C[5]^2) \times C[2] \times 3$   
 [31]  $B[5;4] - (C[5]^2) \times C[3] \times 3$   
 [32]  $B[5;5] - (C[5]^2) \times C[4] \times 3$   
 [33]  $B[6;1] - C[1] \times C[2] \times C[3] \times 6$   
 [34]  $B[6;2] - C[1] \times C[2] \times C[4] \times 6$   
 [35]  $B[6;3] - C[1] \times C[2] \times C[5] \times 6$   
 [36]  $B[6;4] - C[1] \times C[3] \times C[4] \times 6$   
 [37]  $B[6;5] - C[1] \times C[3] \times C[5] \times 6$   
 [38]  $B[7;1] - C[1] \times C[4] \times C[5] \times 6$   
 [39]  $B[7;2] - C[2] \times C[3] \times C[4] \times 6$   
 [40]  $B[7;3] - C[2] \times C[3] \times C[5] \times 6$   
 [41]  $B[7;4] - C[2] \times C[4] \times C[5] \times 6$   
 [42]  $B[7;5] - C[3] \times C[4] \times C[5] \times 6$   
 [43] 'BN LONG-RUN EQUILIBRIUM PROBABILITIES'  
 [44]  $\square - B$   
 [45]  $D - 17 \rho_0$   
 [46]  $D[1;1] - B[1;2] + B[1;3] + B[3;1] + B[3;5]$   
 [47]  $D[1;2] - B[3;2] + B[3;3] + B[4;1] + B[4;2] + B[7;2] + B[7;3]$   
 [48]  $D[1;3] - B[2;5] + B[3;4] + B[4;4] + B[4;5] + B[5;3] + B[5;4] + B[6;1]$   
      $+ B[7;4] + B[7;5]$   
 [49]  $D[1;4] - B[1;4] + B[1;5] + B[5;1] + B[5;5] + B[6;2] + B[6;3] + B[6;4]$   
      $+ B[6;5]$   
 [50]  $D[1;5] - B[2;1] + B[2;2] + B[4;3] + B[5;2] + B[7;1]$   
 [51]  $D[1;6] - B[2;3] + B[2;4]$   
 [52]  $D[1;7] - B[1;1]$   
 [53] 'LONG-RUN EQUILIBRIUM PROBABILITIES FOR STATES WITH TOTALS'  
 [54] 'OF 0, 4, 8, 12, 16, 20, 24 GUNS IN POSITION'  
 [55]  $\square - D$   
 [56]  $E - (D[1;2] \times 4) + (D[1;3] \times 8) + (D[1;4] \times 12) + (D[1;5] \times 16) + (D[1;6] \times 20)$   
      $+ (D[1;7] \times 24)$   
 [57] 'LONG-RUN EXPECTATION OF GUNS IN POSITION'  
 [58]  $\square - E$   
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